Journal of Mathematics Teacher Education
https://doi.org/10.1007/s10857-024-09661-3

# Open mathematical tasks conceived, designed, and reflected upon by preservice elementary teachers

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Accepted: 9 October 2024


## Abstract

As part of the research on mathematical tasks, the literature on the advantages of using open tasks has been steadily growing. However, limited research exists on the types of open mathematical tasks considered by elementary preservice teachers and their abilities to create those tasks. This study explores the preconceptions of open mathematical tasks held by 56 preservice elementary teachers, as reflected in a survey where they described the characteristics and design of such tasks. The study also examines their reflections on implementing open mathematical tasks in one-on-one interviews with elementary students. Our analysis found that elementary preservice teachers described characteristics of open mathematical tasks that match aspects identified in the literature. They also showed the ability to create an open task to a certain degree. However, the open tasks they designed were predominantly procedural and focused on a few aspects of task openness. This finding indicates that the preservice teachers prefer a narrower spectrum of openness when creating open tasks compared to their preconceptions. Preservice teachers also reported challenges they faced when implementing the open tasks with elementary students. These findings suggest the need for mathematics teacher education to provide opportunities where future teachers design a broad spectrum of open mathematical tasks, while addressing on-the-ground challenges in classroom settings.

**Keywords** Open tasks · Teacher conceptions · Task design · Teacher reflection · Preservice teacher education

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Published online: 29 October 2024

## Introduction

In mathematics classrooms, teaching and learning occur through interactions among teachers and students while working on tasks that address specific mathematical content. Thus, how tasks are set up and enacted in the teaching and learning environments significantly impacts students' development and gains in learning (Cohen et al., 2003; Smith & Stein, 1998). As a facet of research on mathematical tasks, the literature addressing the benefits of using open tasks has been continuously accumulating (e.g., Boaler, 1998; Cai et al., 2015; Klein & Leikin, 2020; Pehkonen, 1995; Schoenfeld, 1985). The term, *open task,* is somewhat elusive as different studies refer to different constructs (Yeo, 2017). It is generally assumed that open tasks promote divergent thinking that draws on different skill sets and concepts, a growth mindset by focusing on the learning process instead of just the final answer, and students' engagement (Bingölbali & Bingölbali, 2020; Boaler, 1998).

While prior research offers valuable insight into the benefits of using open tasks, some studies report that their use is infrequent in both intended and enacted curricula and that teachers' familiarity with open tasks remains constrained (e.g., Bingölbali & Bingölbali, 2020; Klein & Leikin, 2020; Yeo, 2017). The testing-heavy school culture may partly cause resistance to open tasks that promote inquiry approaches, leaving little space for them (Potari et al., 2019). Within the scope of research on open tasks, there is little literature exploring the characteristics of open mathematical tasks developed by preservice teachers (PSTs). In response to the generally assumed benefits and challenges of open tasks in the literature and considering the scarcity of research involving PSTs, this study examines elementary PSTs' preconceptions and development of open mathematical tasks. Additionally, this study examines PSTs' reflections on their experiences of implementing open mathematical tasks in one-on-one interviews with elementary students.

This study is designed to provide a space for PSTs to share their insights about what constitutes an open mathematical task and how it should be structured. By fostering open discussions instead of providing strict instruction, our goal is to gather valuable insights for mathematics teacher educators on how PSTs initially conceptualize, develop, and reflect on open tasks. Consequently, the nature of this study is exploratory and does not aim to impose any definitive stance on the use of open mathematical tasks. Instead, it seeks to deepen mathematics teacher educators' understanding of PSTs' preconceptions of open mathematical tasks and bring up discussions around teacher preparation. The following research questions (RQs) guided our inquiry:

RQ 1. What preconceptions do PSTs bring to a mathematics methods course about open mathematical tasks?

RQ 2. What types of tasks do PSTs pose when asked to develop open mathematical tasks?

RQ 3. What aspects do PSTs address to demonstrate the openness of mathematical tasks when asked to develop open mathematical tasks?

RQ 4. What reflections do PSTs take away from their experiences of using the developed open mathematical tasks in the one-on-one clinical interview with elementary students?

In this study, a "mathematical task" broadly refers to a problem, a series of problems, and an extended activity that address target mathematics content situated between teaching, learning, and assessment (Smith & Stein, 1998). For this study, an "open mathematical task" also broadly refers to a non-closed task as opposed to a closed task with a clearly defined task goal, determined solution pathways, approaches, and responses (e.g., Pehkonen, 1997; Sullivan et al., 2000). The reason for adopting these broad definitions is that there is a vague distinction between the terms that are often interchangeably used in the research literature (e.g., problem versus task, open task versus open-ended task). These subtle distinctions among the terms may minimally serve to capture this study's main point of interest.

## Theoretical framework

Our study is built on the theoretical framework developed by Liljedahl et al. (2007) regarding how to design a "good" mathematical task. They defined that the development of mathematical tasks includes four phases: *predictive analysis, trial, reflective analysis, and adjustment.*

Predictive analysis examines how a task designer approaches a mathematical task before they have experience of studying it within a teacher education context. This phase may reveal PSTs' prior experience with solving a mathematical task and their perception of the characteristics of a mathematical task. The second phase is a trial period. A task designer tries out the developed task in a classroom context (e.g., a whole-class, small group, and individual clinical interview). This trial phase is an avenue to test out whether a task introduces mathematical ideas to students meaningfully (i.e., mathematical affordance) as well as how this task can operate within a classroom setting (i.e., pedagogical affordance).

After the trial, the reflective analysis phase follows. The task designer who tried out their task is now reflectively analyzing both affordances and limitations of the task. Their reflection may entail the gap between the intended task and the task that was actually implemented. One example can be to what extent a mathematical task can be meaningfully implemented with students. This opportunity to reflect on the task leads to the adjustment phase. During this phase, a task designer refines their task. These four phases can be reiterated, and each iteration can deepen the task designer's understanding of good mathematical tasks.

In this study, our focus is on the first three phases—predictive analysis, trial, and reflective analysis. This approach will illuminate the conceptions PSTs bring to a mathematics method course (RQ 1), the types and aspects of open mathematical tasks PSTs develop (RQs 2 and 3), and the affordances and limitations PSTs take away from the implementation of open task (RQ 4). Figure 1 shows the theoretical framework of this study.

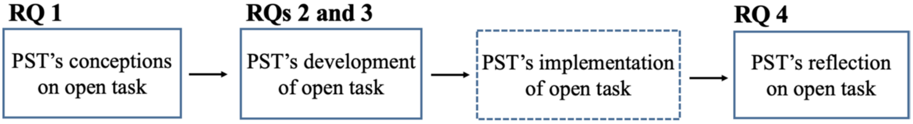



**Fig. 1** Theoretical framework of this study

## Related literature

### What is (not) an open mathematical task?: types and characteristics

Varying degrees of openness or closedness of learning tasks are understood as the opposite ends of a spectrum or different places in a multi-dimensional space (e.g., Black et al., 2004; Stroup et al., 2007). A common consensus is that despite the ambiguity within open tasks, the differences between open and closed tasks are discernible.

A closed mathematical task has a clearly defined task goal,[1] routinized solution pathways, and predetermined answers (e.g., Becker & Shimada, 1997; Pehkonen, 1997; Sullivan et al., 2000). Closed tasks usually intend to offer students opportunities to practice procedural skills based on their prior instruction. These tasks often form a substantial portion of curriculum materials, such as textbooks and enacted instruction (e.g., Bingölbali & Bingölbali, 2020; Boaler, 1998; Klein & Leikin, 2020; Yeo, 2017). For example, a closed task involves calculating the volume of a box when length is 2 cm, width is 5 cm, and height is 6 cm. In contrast, an open task entails creating "an open box using a given vanguard sheet so that it has the biggest possible volume" (Yeo, 2017, p. 184).

While closed tasks have their own purposes and merits, many educators have been concerned about overreliance on this type of task in teaching and learning mathematics due to its prescriptive nature that does not promote divergent thinking. Educators suggest taking advantage of open tasks in response to this dissatisfaction with closed tasks. However, in the extant research literature, the types of open tasks are described from multiple perspectives with various names (e.g., Bingölbali & Bingölbali, 2020; Boaler, 1998; Klein & Leikin, 2020; Stroup et al., 2007; Yeo, 2017), and there are no clear-cut boundaries between them.

Some studies, particularly those exploring creativity and divergent thinking, value open tasks for their contribution to new knowledge construction by connecting various conceptual and procedural aspects of mathematics in a more heuristic way than closed tasks can (e.g., Klein & Leikin, 2020; Kwon et al., 2006; Silver, 1997). However, it should also be noted that "mathematical problem-solving and creativity need not be restricted solely to open-ended problems" (Bokhove & Jones, 2018, p. 302). In other words, understanding the quality of tasks by drawing dichotomous boundaries around their forms (i.e., open versus closed) on a one-dimensional spectrum is not suitable for addressing the complexities involved in mathematical tasks. With this caution in mind, reviewing prior studies helps to understand the range of open tasks and how their non-routine, non-prescriptive nature contrasts with the routinized and prescriptive closed tasks.

Klein and Leikin's (2020) study associates solving open mathematical tasks with a creativity-directed activity because "it promotes and requires mental flexibility and provides multiple opportunities for the production of original ideas" (p. 350). The authors classified such open tasks into four categories: (a) tasks that can be solved with *multiple strategies* that lead to the same solution, (b) tasks that lead to *multiple outcomes*, (c) *investigation* tasks that can be approached in different ways and lead to different discoveries at the end, and (d) tasks that ask students to invent *sorting* criteria that lead to different sorting outcomes (e.g., A set of attribute blocks can be grouped in various ways, such as

[1] The goal indicates the desired result that students are expected to answer. Thus, the goal in this study is different from a lesson goal, learning goal, or learning objective.

shape, size, color, and thickness). This study suggests that open mathematical tasks can be categorized into different types and problem-solving strategies.

Similarly, in developing a framework to assess the openness of tasks, Yeo (2017) examined the representative types of open tasks. He juxtaposed open tasks with closed procedural tasks, which refer to routine practice problems that focus on procedural skills. The open tasks include problem-solving tasks (a challenging and non-routine task that students have not been exposed to before), investigative tasks (a task that asks students to discover the underlying mathematical structures), and real-life tasks (a task involves learning and applying mathematics in/to real-life situations).

Likewise, there is a wide range of open task types that prior studies classified, showing different viewpoints, while there are some overlaps. Researchers noted that the types of open tasks are meant to be representative to explain the characteristics of open tasks instead of being exhaustive. This leads to the first research question of the present study, which examines the types of tasks PSTs conceive to be open. In this study, the types of tasks proposed by PSTs are first inductively examined and then compared with those reported in the prior studies in the analysis process.

## What constitutes the openness of tasks?

Several prior studies have discussed the different elements of tasks that could open space for problem solvers in terms of *depth* and *breadth*. While it is still vague regarding what constitutes the openness of tasks, these studies offer some salient elements of open tasks. Bingölbali and Bingölbali (2020) analyzed the openness of mathematical tasks by dividing each task into three segments (beginning, intermediary, and end). In the beginning segment, the authors checked out whether the task had one or multiple entry points available. For the intermediary segment, tasks were examined in terms of solution methods. The task was considered closed when there is one solution method and open when multiple solutions exist. Because any mathematical task can be solved with different approaches, Bingölbali and Bingölbali (2020) regarded the task with no explicit reference to the solution strategies in the task statements as one solution task. Only the tasks with explicit requirements for multiple solution methods were coded as open-intermediary tasks (i.e., open method tasks). The end segment refers to the outcomes of the task (one correct outcome vs. multiple outcomes).

The analytical framework proposed by Stroup et al. (2007) is similar to the one developed by Bingölbali and Bingölbali (2020) in considering the number of solution methods and solutions (interchangeable with "answers" in our study). However, a key difference is that Stroup et al. (2007) incorporated the characteristic of task goal (e.g., fitting with data and task goal) in determining the types of open tasks. In the development of a taxonomy of generative activities, Stroup et al. (2007) used the terms, *pathways* (i.e., intellectual and/or behavioral routes for arriving at a given endpoint) and *endpoints* (i.e., outcomes created by learners). They separated "multiple-pathway-agreed-upon-endpoint tasks" (p. 374) from the nominally generative activities that a task with a single pathway and endpoint (e.g., $2x+3x=5x$). Then, they proposed five kinds of open tasks considering pathways, endpoints, and the task goal. Those open tasks include: (a) a task with multiple pathways and an agreed upon endpoint (e.g., find three functions that are the same as $f(x)=5x$), (b) a task where multiple pathways and endpoints are supposed to fit with data (e.g., create a variety of mathematical models, use them to produce model outcomes, and find goodness of fit), (c) a task that features multiple pathways and endpoints, with its goal being to conform to design specifications rather than data (e.g., create a mathematical task that can generate a variety of problem-solving approaches and discuss why some approaches are more functional than the others), (d) a task that engages students in role-playing activities (e.g., students pretend to be a driver. They locate themselves in a Cartesian coordinate. Then, they relocate themselves by moving to a place where their y-value is two-times their $x$-value), and (e) a task that focuses on exploration of kind and quality of pathways (e.g., what kind of reasoning allows us to claim that "$5x + 1x$" as being the "same" as "$3x + 3x$"?). This study implies that the spectrum of an open mathematical task is not simply determined by how many pathways or endpoints exist, but also by the goal of the task.

While Stroup et al. (2007) and Bingölbali and Bingölbali (2020) examine how the key segments of tasks allow for open space, such as creativity, flexibility, and multiple problem-solving strategies, Yeo (2017) points out that educators mean different constructs when speaking of the same segments of open tasks. He problematized that this situation could generate confusion in research or teaching related discussion. To address this issue, Yeo (2017) proposed a framework that consists of five task variables: "goal," "method," "complexity," "answer," and "extension." While each task variable can be open or closed, Yeo's framework recognizes that a variable can also have multiple dimensions of openness. For instance, concerning the "method" variable, a task is deemed to possess a closed method if only one method is available or if the method solely involves routine application of known procedures. Otherwise, the task is characterized as having an open method.

The open method tasks can be further categorized into *well-defined* vs. *ill-defined* methods and *task-inherent* vs. *subject-dependent* methods. A well-defined method indicates that it is feasible to instruct students in a method that will consistently yield the same correct answer. Conversely, an ill-defined method means the same method taught to different students may produce different answers. A task-inherent method refers to openness inherent in the task itself, such as in investigative tasks, where utilizing only one method to derive all correct answers is impossible. On the other hand, a subject-dependent method implies that openness depends on the individuals involved. For instance, problem-solving tasks are expected to be open regarding method. However, if teachers fail to instruct students on discovering alternative solution methods, then the tasks become closed to the students. Therefore, Yeo's analytical framework urges careful examination in deciding the openness of mathematical tasks by accounting for multiple dimensions of each task variable.

There are some overlaps between these studies (Bingölbali & Bingölbali, 2020; Stroup et al., 2007; Yeo, 2017), but they differ in focus elements and sophistication. This suggests that tasks can have varying degrees of openness regarding depth and breadth rather than exclusively open or closed. Therefore, previous literature justifies using a combination of a priori codes from prior studies (Bingölbali & Bingölbali, 2020; Stroup et al., 2007; Yeo, 2017) and highlighting emergent codes derived from the data in the study to analyze the types and characteristics of open mathematical tasks proposed by research participants. The method section provides a detailed description of the analytical framework that we used for the present study.

Meanwhile, it is worth noting another framework that highlights a different aspect of mathematical tasks such as cognitive demands explained by Stein and Smith (1996), which refers to "the kind of thinking processes required to solve the task as presented by the teacher" (p. 461) Stein and Smith (1998) categorized cognitive demand to four levels: 1) memorization (i.e., committing or producing previously learned facts, rules, formulae, or definitions), 2) procedures without connections (i.e., focusing on algorithms based on prior instruction or experience), 3) procedures with connections (i.e., using procedures for the purpose of developing deeper levels of understanding of mathematical concepts), and 4) doing mathematics (i.e., requiring non-algorithmic thinking and exploration of the nature of mathematical concepts, processes, or relationships).

The level of cognitive demand is not inherently linked to the openness of the task (e.g., assuming that open tasks are always high cognitive demand tasks). For example, the task "Find three equivalent fractions of 3/5" is a low cognitive demand task if students already familiar with the rule for finding equivalent fractions by multiplying or dividing both the numerator and denominator of a fraction by the same number. Nevertheless, this is an open task with multiple possible answers. Conversely, the task "Imagine you are at a party and a cake is cut into nine equal pieces. [You take one of those nine equal pieces.] Two people show up to the party late, and you decide to share your piece of cake with them. So, what fraction of the whole cake do the latecomers get together? (Kerrigan et al., 2020, p. 2255)" presents a high cognitive demand for students, as they must make sense of multiple mental processes that underpin fraction knowledge. Students must first partition the whole cake into nine pieces, then subdivide one of those pieces to share among three people. Finally, they must partition the new portion equally into three pieces. However, this task is more closed than tasks involving equivalent fractions, as students are likely to follow the prescribed linear process, and the answer is determinate.

Complexity, one of the five task variables of open tasks defined by Yeo (2017), may be considered similar to Stein and Smith's (1998) notion of cognitive demand. However, these two are still differ. While complexity (Yeo, 2017) determines whether a task is complex based on whether students can close the task, the cognitive demand "refers to the kind of thinking processes entailed in solving the task as announced by the teacher" (Stein et al., 1996, p. 461).

## Studies on teachers' conceptions and performance of open tasks

Much research on open mathematical tasks has been undertaken. Those include characteristics of open tasks (e.g., Bennevall, 2016), students' performance of open tasks (e.g., Cai, 2000), analysis of mathematical tasks in the textbook (e.g., Bingölbali & Bingölbali, 2020; Zhu & Fan, 2006), divergent-convergent thinking and creativity concerning open tasks (e.g., Bennevall, 2016; Kwon et al., 2006), and effects of professional development on teachers' understanding and skills of open tasks (e.g., Klein & Leikin, 2020; Pehkonen, 1999; Zaslavsky, 1995). This section reviews studies on in-service and PSTs' conceptions and performance of open tasks.

Regarding in-service teachers' conceptions and implementation of open tasks, Pehkonen's (1999) project on teachers' conceptions of open tasks found that about half of the participants struggled to articulate a proper definition for open tasks, showing their unfamiliarity with the open task. In the study of Turkish teachers' openness to and evaluation of different solutions to problems, Bingölbali (2011) observed that teachers did not place significant value on tasks with multiple solutions and encountered challenges in grading diverse solutions students provided. Similarly, the study conducted by Nabie et al. (2016) on Ghanaian primary in-service teachers' conceptions and implementation of open tasks uncovered that despite the curriculum policy endorsing the multiple solutions approaches, teachers often strayed from the recommended curriculum guidelines in their practice.

In Klein and Leikin's (2020) study, 44 in-service teachers were asked to pose and solve open tasks and complete a questionnaire regarding their conceptions of posing open tasks and teaching and learning with them. The findings show that multiple strategies tasks were the most used and familiar type of open task. However, the participants reported that they were less familiar with sorting tasks as open tasks. This study implies that similar to in-service teachers, PSTs may be familiar with only certain kinds of open tasks. Recently, Levenson (2022) explored teachers' values on tasks by asking them to choose the task that they believe would offer the most potential for mathematical creativity in the classroom. The findings show that teachers valued the potential for multiple solution methods over other features (e.g., multiple final answers, multiple representations, etc.). In sum, research reports on in-service teachers' unfamiliarity and resistance to incorporating features of open tasks as well as their familiarity and preference for specific elements of open tasks.

Research on PSTs' conceptions and design of open tasks reported that teacher education should offer PSTs opportunities to examine, evaluate, and develop quality mathematical tasks (Isik & Kar, 2012; Lee, 2012; Lee & Hwang, 2022). Studies suggest that providing PSTs with those opportunities can bolster their acquisition of mathematical and pedagogical knowledge (e.g., Lee & Lee, 2021; Thanheiser et al., 2016). However, not many studies examine the PSTs' conception and development of open tasks, which is the driving force behind our study.

Spiliotopoulou and Potari's (2002) study is one of a few examples that studied PSTs' approach to open tasks. They investigated PSTs' views as learners, designers, and users of open mathematical tasks. The authors noted that PSTs focused more on the phenomenological features of the problem rather than deeper aspects of the problem, such as the required thinking processes behind the mathematics problem. The PSTs also showed a limited understanding of the role of open tasks in teaching and learning mathematics. When implementing the task they designed, PSTs seemed to have gained a certain level of understanding about the meaning and role of open tasks in the classroom. However, their study examined PSTs' approach to open tasks based on their self-reported claims.

Bragg and Nicol (2008) examined elementary PSTs' experiences of posing open-ended mathematical tasks. They analyzed self-reported accounts of the PSTs' approach to creating such tasks, the difficulties encountered during the task design, and the subsequent influence of this task on their development as educators. The results showed that this experience allowed PSTs to examine their previously held perspectives on mathematical tasks and provided awareness about what kinds of tasks can provide good learning practice, including problem posing. Paredes et al. (2020) examined changes in openness, among other characteristics of tasks (e.g., authenticity and cognitive domains). The PSTs in their study created, refined, and reflected on realistic tasks through a series of course activities, and the authors reported the PSTs' increased level of task openness over time. In Paredes et al.'s (2020) study, however, the term "openness" was narrowly defined regarding the number of possible answers, distinguishing between tasks with multiple correct answers and those with only one correct answer. This study also gathered data within the university course setting without involving task implementation in the school environment.

### The current study

Studies on PSTs' conceptions and performance regarding open tasks are limited in numbers. Most prior studies (e.g., Bragg & Nicol, 2008; Paredes et al., 2020; Spiliotopoulou & Potari, 2002) also have limitations in that they mainly focused on the task design rather than comprehensively examining PSTs' conceptions, task design, and reflection process (Liljedahl et al., 2007). Thus, existing literature cannot ensure the alignment between PSTs' conceptions, the tasks they designed, task implementation, and their reflection on the task implementation. Furthermore, most studies were limited to the university classroom setting (e.g., Paredes et al., 2020). Thus, the present study aims to contribute to the field of mathematics teacher education by examining the open mathematical tasks conceived and created by PSTs and offering opportunities for PSTs to reflect on their teaching experiences in the field (i.e., elementary school classrooms).

## Method

### Participants and context

The participants in this study included 56 PSTs enrolled in an elementary mathematics methods course at a midwestern university in the USA, and the first author was their instructor. These PSTs had completed two mathematics content courses focusing on number theory and geometry before this methods course. They also took several general educational foundation courses concentrating on instructional design, assessment, and classroom management, which were not specifically tailored to mathematics. Although it was not the sole focus, some discussions on differentiation (Tomlinson, 2000), understanding by design (McTighe & Wiggins, 2005), and universal design for learning (Novak, 2022) occurred in their previous courses. The PSTs in this study also had some field experiences throughout the program at local schools for participatory observation and limited levels of instructional experiences under the supervision of cooperating teachers.

As part of the course activities, the PSTs developed a minimum of three mathematical tasks to elicit and interpret elementary students' mathematical understanding in the one-on-one clinical interview setting. The PSTs were asked to identify one of these three tasks as an open task. They were also asked to prepare for a 30-min in-person clinical interview using the tasks they had developed, including an open task. In the preparation of the clinical interview, the PSTs had opportunities to watch videos of other teachers' clinical interviews and explore various talk moves (Chapin et al., 2013). Partner elementary students were selected from the classes of the cooperating teachers, where the PSTs regularly observed and familiarized themselves with the school curriculum and students. However, the opportunities for observing and participating in mathematics classes varied among PSTs, depending on their school visit and observation schedules. PSTs were encouraged to gather written work from partner elementary students for analysis and reflection, but student work was not part of the data for this study.

This study intentionally limited its scope to this condition to focus on PSTs' preconceptions of open tasks by reducing the complexity associated with other variables. After concluding the semester, we began analyzing de-identified written work samples. These samples included PSTs' descriptions of the characteristics of open mathematical tasks, the open tasks they created, and their reflections on using the open task with elementary students. At the time of data collection, the PSTs had reviewed Common Core State Standards, which outline the knowledge and skills students are expected to achieve at each grade level in English Language Arts and Mathematics (NGA & CCSO, 2010) in the US context. They also explored some teaching practices, including eliciting and interpreting student thinking (TeachingWorks, 2024), clinical interviews (Ginsburg, 2009), and number talks (Parrish, 2011).

## Data sources

This study is a part of a larger study investigating PSTs' approaches to task development and modification. As this study aims to investigate PSTs' conception, development, and reflection of open tasks based on their prior knowledge and experience, we refrained from providing explicit instruction on open-ended tasks, such as definitions or methods supporting their development of open mathematical tasks.

Along with the theoretical framework (Fig. 1), three data sources were collected in three distinct phases. In the first phase (beginning of the semester), for RQ 1, an online survey asked participants to describe characteristics they consider "open mathematical tasks." In the second phase, for RQs 2 and 3, each PST completed a written report in response to the following directions in the classroom: "Develop a problem (or modify an existing problem) that you consider an open task to be used for the assessment of students' mathematical understanding."

The required components the PSTs included in the report for RQs 2 and 3 were as follows: (a) target grade level and relevant standards, (b) the task to be presented to students, (c) anticipated students' solution methods, (d) acceptable answers, and (d) explanation of why this task was considered open. Then, these open task items were utilized along with other tasks that they developed to interact with their partner elementary students in the one-on-one interview setting during the practicum. The de-identified reports from 56 PSTs were analyzed after the semester ended.

In the third phase, for RQ 4, PSTs were asked to submit their reflections on their experiences of using the open task with elementary students by responding to the following prompts:1) How was your experience using the open task with your elementary partner student?; 2) What were your biggest takeaways from your experience?" The PSTs' reflections were gathered through an anonymous online survey. A total of 40 responses were collected and summarized.

## Data analysis

For RQ 1 on PSTs' preconceptions of open tasks and RQ 3 on the openness of the tasks posed by PSTs, this study employed a hybrid approach of a priori and emergent codes (Mayring, 2014). Based on research literature (Bingölbali & Bingölbali, 2020; Klein & Leikin, 2020; Stroup et al., 2007; Yeo, 2017), first, we listed hypothesized a priori codes. Then, we combined, collapsed, expanded upon, and revised the priori codes to create a framework that would be used to analyze our data from the survey and PSTs' written reports (see Table 1). The emergent codes drawn from such data are reported in the findings section (see Table 3).

**Table 1** Analytical framework (adapted from Bingölbali & Bingölbali, 2020; Klein & Leikin, 2020; Stroup et al., 2007; Yeo, 2017)

| Category | Openness | Description | Example |
|---|---|---|---|
| Goal (The desired result of the task[a]) | Closed | There is a specific goal in the task statement | Solve the quadratic equation $x^2+2x-3=0$ |
| | Open; ill-defined | No specific goal is in the task statement; students need to choose their goals | Powers of 3 are $3^1$, $3^2$, $3^3$, $3^4$, … Investigate |
| | Open; well-defined | No specific goal is in the task statement; the task statement includes a task requirement, yet keep the goal open | Powers of 3 are $3^1$, $3^2$, $3^3$, $3^4$, … Find as many patterns as possible |
| Entry (A place to start the task) | Closed | The task has a single choice in starting | Solve $23+54$ |
| | Open | The task allows students to choose a specific problem, condition, or context for multiple entry points | Choose two two-digit numbers and add them. What is the sum? |
| Method (A particular pathway to accomplish a task goal) | Closed | In the task statement, students are not explicitly told to solve using multiple strategies | There are four pencils in each of three glasses. Find the total number of pencils |
| | Open; specified | The task contains an explicit and specific requirement for solving the problem in multiple ways | There are four pencils in each of three glasses. Find the total number of pencils through addition and skip counting |
| | Open; general | The task contains a general requirement for solving the problem in multiple ways | There are four pencils in each of three glasses. Find the total number of pencils using different strategies |
| Answer (The outcome of the task) | Closed | The answer is determinate | Solve the quadratic equation $x^2+2x-3=0$ |
| | Open; objective | The answer is indeterminate and clearly defined to be either right or wrong | Choose two two-digit numbers and add them. What is the sum? |
| | Open; subjective | The answer is indeterminate, and there is no right or wrong answer | Design a playground for the school |
| Complexity (Level of difficulty) | Closed | The task is simple enough for students | Solve the quadratic equation $x^2+2x-3=0$ |
| | Open; subject-dependent | The task is complex for students, but the teacher can provide enough scaffolding to close the task | Powers of 3 are $3^1$, $3^2$, $3^3$, $3^4$, … Investigate the pattern |
| | Open; task-dependent | The task is complex for students but it is inherently not possible to provide enough scaffolding to close the task | Design a playground for the school |

**Table 1** (continued)

| Category | Openness | Description | Example |
|---|---|---|---|
| Extension (The possibility of extending the scope of the task) | Closed | The task has no room for extension | Powers of 3 are $3^1, 3^2, 3^3, 3^4, \ldots$ Investigate |
| | Open; subject-dependent | The tasks are not expected to be extended by most subjects (e.g., students). The extension is up to the subjects' decisions | 100 participants shake hands once with each of the other participants. Find the total number of handshakes |
| | Open; task-dependent | The tasks are expected to be extended by independent of the subjects' decisions | Design a playground for the school |

[a] The "goal" in our analytical framework pertains to the mathematical task, specifically whether a specific goal (the desired result) is stated in the task statement/directions. Thus, the goal in this analytical framework is different from a lesson goal, learning goal, or learning objective

**Table 2** Examples of task analysis process

| Task | Analysis |
|---|---|
| Solve $5 \times 13$ using as many different ways as possible. (ID#32) | Goal: Closed (A goal [solving $5 \times 13$] is specified in the task statement)<br>Entry: Closed (One problem is given)<br>Method: Generally open (It is asked to solve the problem in multiple ways)<br>Answer: Closed (There is one correct answer)<br>Complexity: Closed (Assuming that it is simple enough for the student)<br>Extension: Closed (It cannot or should not be extended)<br>Task type: Procedural tasks (Practicing a simple multiplication) |
| Let's plan a class party. Food and drink to share, activities to do, and prizes to give away. Propose your estimated budget. (ID #55) | Goal: Closed (A goal [proposing the estimated budget] is specified in the task statement)<br>Entry: Open (The task allows students to choose a specific condition or context)<br>Method: Closed (While the problem could be solved in multiple ways, in the task directions students are not explicitly told to solve using multiple strategies)<br>Answer: Objectively open (The answer is indeterminate but the reasonableness of the estimation can be determined based on the data provided.)<br>Complexity: Task-dependent open (The task is inherently complex and is not possible to provide enough scaffolding to close the task)<br>Extension: Task-dependent open (The task is expected to be extended)<br>Task type: Authentic real-life tasks (Students need to research relevant information such as prices for food, drink, prizes, and materials for the activities before calculating and proposing their estimated budget) |

For RQ 2 about the types of open tasks, the following categories were used by adapting prior studies. This is not meant to be an exhaustive list of task types, but rather to provide a snapshot of PSTs' ideas in this study:

- *Procedural tasks* involve the practice of procedures or computations that students have previously learned, so students often know right away how to approach the solution (Lester, 1980, as cited in Yeo, 2017).
- *Pure-mathematics investigative tasks* ask students to discover the underlying patterns or mathematical structures without being given specific goals or outcomes (Becker & Shimada, 1997, as cited in Yeo, 2017; Orton & Frobisher, 1996, as cited in Yeo, 2017).
- *Sorting tasks* ask students to invent sorting criteria for given mathematical objects, and different sorting criteria lead to different sorting outcomes (Klein & Leikin, 2020).

- *Authentic real-life tasks* allow students to learn and apply mathematics in real-life situations where students have to do some research to solve a problem. Usually, students need to conduct some research to gather necessary information to solve a problem. Typical "standard problems" in the form of story problems, which can be modeled or solved through the straightforward application of one or more arithmetic operations with the given numbers, are not considered real-life tasks in this analysis (Lee, 2012; Moschkovich, 2002; Yeo, 2017).

The researchers independently reviewed all posed problems, sorted them into the five preset categories, and then compared the results. There was no discrepancy in the results of the analysis between coders. Table 2 shows an example of the task analysis process.

To answer RQ 4 about the PSTs' reflection on the experience of implementing their own open tasks in the one-on-one clinical interview setting, we analyzed the PSTs' reflective statements upon completing the tasks with elementary students. Forty out of 56 PSTs submitted their reflections. Some PSTs offered multiple opinions. For example, for the second question on the biggest takeaways, some mentioned more than one takeaway, while some provided generic statements regarding open tasks rather than specific details about their experiences with open task implementations. Nevertheless, their reflective statements included their feelings, thoughts, and challenges regarding task implementations, which were not addressed explicitly in their work samples. We assigned descriptive labels to their reflective statements (e.g., unexpected student reactions as challenges) to find repetitive themes arising from data (Mayring, 2014). The first two researchers coded a random sample of about 25% of the PSTs' responses, and the concordance between the two coders was about 92%. Then, the two coders coded conjointly for the rest of the data to address any coding discrepancies. Lastly, we calculated the number of responses in each category to find overall tendencies.

## Findings

### Findings from phase 1: PSTs' preconceptions

For RQ 1, PSTs were asked to describe the characteristics of what they conceived as open mathematical tasks at the beginning of the semester. All PSTs proposed multiple aspects, and each different idea was considered as one response. These responses were first sorted based on the categories listed in Table 1 (i.e., goal, entry, method, answer, complexity, and extension). Then, we revised the initial categories (see Table 3) because some emergent codes from the PSTs' responses did not fit in the a priori framework. Table 3 summarizes PSTs' preconceptions of open tasks.

Four categories in the PSTs' responses correspond to the a priori categories listed in Table 1. (i.e., answer, method, entry, and complexity). The most frequently mentioned category by PSTs was the open answer ($n=35$, 63%, either no definitive answer or multiple answers), followed by the open method ($n=28$, 50%, multiple strategies). About 23% ($n=13$) of PSTs highlighted either the openness in students' representational choices or opportunities for students' problem posing. About 11% ($n=6$) of PSTs addressed that open tasks need to be complex.

**Table 3** PSTs' preconceptions on open mathematical tasks ($N=56$)

| Category ($n$, %) | Subcategory ($n$, %) | Example |
|---|---|---|
| Answer (35, 63%) | No definitive answers (21, 38%) | "Tasks or problems that I consider open do not have a definite answer." |
| | Multiple correct answers (14, 25%) | "To me, "open" means there can be multiple different correct answers." |
| Method (28, 50%) | | "It could be a problem that can be solved in more than one way." |
| Entry (13, 23%) | Students' choice of representation (9, 16%) | "Another thing we can do is allow students to submit their thinking and reasoning in various forms such as drawing, writing, or whatever makes sense to them." |
| | Students' opportunities for problem posing (4, 7%) | "Asking students to create their own problem that goes along with certain lessons, then have them solve it to see their thinking." |
| Complexity (6, 11%) | | "They are usually difficult and take a bit longer to solve, and incorporate various concepts that students are learning." |
| Requirements of the task (31, 56%) | Requiring explanations (18, 32%) | "Asking students to explain their reasoning when answering a question." |
| | Requiring collaboration (7, 13%) | "Having students brainstorm ideas [with their peers] of how to come to a solution, problem-solving to understand the next step." |
| | Requiring discussion/debate (6, 11%) | "An "open" problem could involve an opinion, which could be a debate. Maybe it's led by a simple question like "Which is the best way to…?" |
| Formats of the task (18, 32%) | Game (8, 14%) | "I think "open" problems or tasks include games that make the learning visual and more hands-on." |
| | Real-life connection (4, 7%) | "Get a more real-world perspective on math, and students can apply real-world situations to the problem" |
| | Math talks/Number talks (5, 9%) | "A type of task/problem considered "open" is number talks." |
| | Non-multiple choice problems (1, 2%) | "Open problems should not be multiple choice problems." |
| Other/Unspecified (8, 14%) | | "This can show a student's understanding of a math concept." |
| Characteristics of closed task (2, 4%) | | "This is a question or task that only has one known answer or way to do something (e.g., what is 3 + 2? what is 16/3?)" |

The total number of responses exceeds the number of participants because some PSTs provided multiple responses. The frequencies indicate the number of PSTs whose responses included the category

The newly created four categories (i.e., requirements of the task, formats of the task, other/unspecified, characteristics of closed task) capture the responses that did not exactly fit in the a priori categories. These responses had commonalities in providing fairly short descriptions of the nature of the tasks. For example, about 56% ($n=31$) of PSTs highlighted task requirements (e.g., explanations, collaboration, and discussion), and 13% ($n=18$) of responses simply named the formats of tasks (e.g., game, real-life connection, number talks, and non-multiple-choice problems) without describing the characteristics of open tasks. Additionally, two PSTs ($n=2$, 4%) stated the characteristics of closed tasks, not open tasks. Although some inferences could be made to the a priori categories (e.g., a real-life connection might refer to tasks with potential for extension, or number talks might be associated with multiple solution strategies), the separate categorization allowed us to acknowledge various perspectives that PSTs initially held regarding what open task means to them.

## Findings from phase 2: PSTs' task design

Each PST posed one open task. The 56 tasks created by PSTs were examined regarding task types (RQ 2) and openness (RQ 3).

### Task types

The four types of open tasks informed by prior studies (e.g., Bingölbali & Bingölbali, 2020; Klein & Leikin, 2020; Stroup et al., 2007; Yeo, 2017) were used to analyze the PSTs' work: (a) procedural tasks, (b) pure-mathematics investigative tasks, (c) sorting tasks, and (d) authentic real-life tasks. This analysis was based on the researchers' judgment based on our analytical framework, not what PSTs claimed. For example, PSTs claimed that the following two examples were real-life tasks because the contexts relate to students' daily lives.

- Example 1: I have 24 books. Of the 24 books, 6 are fiction and 18 are non-fiction. Describe the relationship between these numbers in as many different ways as possible. (ID #10)
- Example 2: Let's plan a class party. Food and drink to share, activities to do, and prizes to give away. Propose your estimated budget. (ID #55)

However, we classified Example 1 as a procedural task because the context did not provide sufficient room for research study and application of mathematical knowledge; instead, the problem can be solved in a typical and routinized manner (e.g., finding the ratio of two numbers). We categorized Example 2 as an authentic real-life task because the student needed to research the appropriate items, their prices and quantities, and apply their mathematical knowledge and skills to estimate the most reasonable budget.

Of the 56 tasks submitted by PSTs, one authentic real-life task (ID#55 above) and two sorting tasks were identified. The remaining 53 (95%) were procedural tasks, and there were no qualified examples for pure-mathematics investigative tasks. The two sorting tasks took the form of a mathematics routine called *Eliminate It,* asking students to find sorting criteria for four items so that only one did not belong to the same category as others.

**Table 4** PSTs' preconceptions on open mathematical tasks and their justifications for open task development ($N=56$)

| Category | Preconceptions on open tasks, *n* (%) | Justification for open task development, *n* (%) |
|---|---|---|
| Answer | 35 (63%) | 36 (64%) |
| Method | 28 (50%) | 32 (57%) |
| Entry | 13 (23%) | 31 (55%) |
| Complexity | 6 (11%) | 0 (0%) |
| Requirements of the task | 31(56%) | 0 (0%) |
| Formats of the task | 18 (32%) | 1 (2%) |
| Other/Unspecified | 8 (14%) | 0 (0%) |
| Characteristics of closed task | 2 (4%) | 0 (0%) |

The total number of responses exceeds the number of participants because some PSTs provided multiple responses. The frequencies indicate the number of PSTs whose responses included the category

Although not necessarily related to task openness, it is worthwhile to examine the types of tasks that PSTs developed from another perspective: the level of cognitive demand of the task (Stein & Smith, 1998). None of the tasks developed by PSTs were identified as *memorization* tasks. Three tasks (one authentic real-life task and two sorting tasks) could be placed in the *doing mathematics* category because it requires non-algorithmic thinking and has the potential to explore various mathematical concepts. Among the remaining 53 tasks, six tasks could be classified as "procedures without connections" because they presented computational problems, and the directions only stated to "solve the problem." However, when considering all the components in PSTs' report, including anticipated students' solution methods, acceptable answers, and explanations of why this task was considered open, PSTs indeed anticipated various solution methods using various properties of operations. Thus, while the six tasks might seem to require the use of known algorithms, PSTs expected that students would not just perform computations mindlessly. The rest of the tasks (47 tasks) could be classified as *procedures with connections* exhibiting relevant characteristics.

## Claimed justifications of task openness by PSTs

When examining the claimed justifications provided by the PSTs regarding why the tasks they created were considered open, we observed that they focused on fewer aspects compared to the categories identified in their preconceptions of open tasks (refer to Table 3). Specifically, they mainly emphasized open entry (a place to start the task), method (a particular pathway to accomplish a task goal), and answer (the outcome of the task). Twenty-two PSTs justified their open tasks by referring to a single category, while the remaining 34 PSTs explained the openness of their tasks by referring to two or more categories.

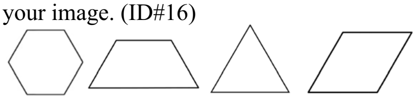
Table 4 compares the categories and frequencies in PSTs' preconceptions of open tasks and the justifications they provided for their task development. Our data showed that only two categories, "answer" and "method" are where PSTs show consistency in what they believe constitutes open tasks and how they justify the task that they created as open. One

**Table 5** Openness of PSTs' developed tasks by three task components ($N=56$)

| Task Component | Openness | Frequency |
|---|---|---|
| Entry | Closed | 26 (46%) |
| | Open | 30 (54%) |
| Method | Closed | 32 (57%) |
| | Open; specified | 2 (4%) |
| | Open; general | 22 (39%) |
| Answer | Closed | 13 (23%) |
| | Open; objective | 42 (75%) |
| | Open; subjective | 1 (2%) |

**Example 1.**

Within the space provided, use the pattern blocks to create a unique image of your choice. Decide your whole (it should be one of the following shapes) and explain the total value of your image. (ID#16)



**Example 2.**

0----------1----------2----------3----------4

Create a fraction addition problem that has an answer indicated by the arrow. Solve your problem. (ID#13)

**Example 3.**

Choose a day and write the month, day, and year. Make as many correct number sentences as you can create using these digits. You can change the order of the digits and can use any mathematical symbols and signs. (ID#56)

**Fig. 2** Examples of open entry tasks

notable difference is in the category of "entry." A higher number of PSTs (31 PSTs, 56%) claimed that they had created a task with open entry, although only 13 PSTs (23%) initially believed open entry as a characteristic of an open task. Additionally, the other five categories that emerged from the initial conception analysis (i.e., complexity, task requirements, task formats, other/unspecified, characteristics of a closed task) did not appear in PSTs' justifications for their task development. This finding indicates a gap between the PSTs' preconceptions of open task and justifications for open task development.

**Table 6** Pathways taken in designing open tasks ($N=56$)

| Entry ($n$) | Method ($n$) | Answer | | |
|---|---|---|---|---|
| | | Closed | Open; objective | Open; subjective |
| Closed (26) | Closed (4) | 3 | 1 | 0 |
| | Open; specified (1) | 0 | 1 | 0 |
| | Open; general (21) | 10 | 11 | 0 |
| Open (30) | Closed (27) | 0 | 27 | 0 |
| | Open; specified (1) | 0 | 1 | 0 |
| | Open; general (2) | 0 | 1 | 1 |

## Examined task openness by researchers

This section presents the researchers' analysis of the openness of the tasks that the PSTs designed according to the analytical framework (see Table 1). However, we did not examine the goal, complexity, and extension of the tasks due to a lack of variety of them. Specifically, all 56 proposed tasks were closed in their goals because all of them had specific goals in the task statements (e.g., "solve," "eliminate," and "propose the estimated budget"). Also, only one PST's real-life task demonstrated task-dependent complexity and extension ("Let's plan a class party: food and drink to share, activities to do, and prizes to give away. Propose your estimated cost." [ID#55]). Thus, we focused on the other three task components (entry, method, and answer), as shown in Table 5.

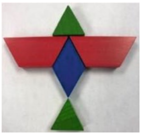
*Entry* Of the 56 tasks, 26 (46%) were closed-entry tasks where only a single choice of work was provided. The remaining 30 (54%) were open entry tasks because students could choose a specific problem, condition, or context to start the task. For the open entry tasks, many PSTs claimed that these tasks could allow students to differentiate and adjust the level of complexity and difficulty. Figure 2 shows some examples of open entry tasks.

*Methods* There were 32 (57%) closed method tasks, two (4%) specified open tasks, and 22 (39%) generally open tasks. Of the 32 closed method tasks, there were discrepancies between what PSTs claimed and the researcher's judgment ($n=10$). Some PSTs ($n=4$) claimed that their tasks were open because they could be solved in multiple ways, and they indeed presented multiple anticipated student approaches (2 to 4 different approaches) in their reports. However, following the guidance of Bingölbali and Bingölbali's (2020) study, we decided to code them as closed method tasks because there was no explicit reference in the task instruction for using different methods (e.g., solve the problem using different strategies). Also, the other six PSTs who proposed multiple entry tasks (e.g., asking students to choose a specific problem, condition, or context to start the task) did not present specific solution strategies in their reports, and there was no reference in the task instruction for using different strategies.

Below is one of the two tasks coded as specified open method tasks. In this example, the PST explicitly required the use of multiple models in the task instruction and claimed that this feature made the task open, along with the fact that students could choose a fraction.

- Show the value of one fraction of your choice using three different fraction models (area, length, and set). (ID#10)

**Table 7** Examples of open task classification

| Example | Classification |
|---|---|
| **Example 1.**<br><br>Eliminate a number that does not belong. Try to find different ways to eliminate numbers.<br><br>15 \| 12<br>24 \| 20 | • Entry: Closed (Numbers are given)<br>• Method: Generally open (Asked to find different ways to eliminate numbers)<br>• Answer: Objectively open (Depending on the student's criteria, the answer is defined to be either right or wrong) |
| **Example 2.**<br><br>Using at least three kinds of pattern block shapes (hexagon, rhombus, trapezoid, triangle), create an animal and find out what fraction of the whole is the head!<br><br>Use my work as an example. (ID#26)<br>I made a bird. If the entire bird is my whole, ___ of the whole are the wings.<br> | • Entry: Open (Students can create different shaped animals to start the task)<br>• Method: Closed (There is no explicit reference in the task instruction for using different methods)<br>• Answer: Objectively open (Depending on the shape the student created, the answer is defined to be either right or wrong) |

The remaining 22 tasks were categorized as generally open method tasks where the task instruction referenced multiple ways but without specific requirements, as shown in the following example (e.g., "solve as many different ways as possible.").

- I have 24 books. Of the 24 books, 12 are fiction and 6 are non-fiction. Describe the relationship between these numbers in as many different ways as possible. (ID#30).

*Answer* Most tasks ($n=42$, 75%) contained objectively open answers, and 13 (23%) had closed answers. Only one task (2%) had subjectively open answers. Half of the closed-entry tasks yielded closed answers, and the other half ended with open answers. Naturally, all open entry tasks had all open answers, and they were all objectively open answers except for one case. Shown below are some examples.

- Objectively open answer task: How can you break up the number $\frac{5}{8}$ ? Are there multiple ways of doing this? Show as many different ways as you can. (ID#37)
- Subjectively open answer task: Let's plan a class party. Food and drink to share, activities to do, and prizes to give away. Propose your estimated cost. (ID#55)

### Patterns in designing open tasks

The preceding section highlighted the PSTs' approaches to making open tasks across three components (entry, method, and answer). Table 6 shows the overall patterns in designing open tasks by examining the pathways through which the three components are managed.

In closed-entry cases ($n=26$), the most frequent approach took the pathways (i.e., intellectual and/or behavioral routes for arriving at a given endpoint from Stroup et al., 2007) of requiring generally open methods ($n=21$, e.g., solve as many different ways as possible). Among those 21 tasks, 11 were designed to yield objectively open answers. In addition, only three tasks were completely closed (closed entry, method, and answer), indicating that 23 out of 26 closed-entry cases used open method and/or open answer. This result shows that most mathematical tasks created by the PSTs have at least one open aspect in terms of the categories of entry, method, and answer.

In open entry tasks ($n=30$), most PSTs asked students to self-pose problems to open the entry, and the design patterns were much simpler than closed-entry tasks. It appears that most PSTs took the pathways of closed methods and objectively open answers (27 out of 30 cases) because there was no explicit reference in the task instruction for using different methods, and the answers could be determined based on the problems students posed. Table 7 shows examples of open task classification.

## Findings from phase 3: PSTs' reflections[2]

This section highlights the repetitive themes that arose from PSTs' reflections on implementing open tasks during the one-on-one clinical interviews with elementary students. In general, PSTs tended to reflect on the challenges that they faced rather than the opportunities available to them.

### Unexpected students' reactions

Some PSTs claimed that their tasks were open because they specifically asked to solve the problems in multiple ways (e.g., Find as many patterns as possible). However, their partner elementary students were not interested in multiple methods once they found the final answer. Also, some students continuously sought confirmation on the correctness of their solution methods from PSTs even though it was said that the problems could be solved in multiple ways. Thus, PSTs felt that their partner students had not been exposed to open tasks in their typical mathematics classrooms frequently. One PST stated, "I wonder if we are asked to do things that are not considered realistic in the real classrooms." PSTs also addressed the need to set up mutually agreed expectations and norms regarding solving open tasks between the teacher and student before implementation.

### Uncertainty in the role of open tasks for challenging students

A couple of PSTs who developed open entry tasks by allowing elementary students to pose and create their own problem noticed that their partner students tended to choose an easier

[2] 40 out of 56 PSTs submitted their reflections on the two questions (see the method section) via an anonymous online survey. Due to the anonymity of the survey, we could not specify PSTs' IDs alongside the excepts they provided.

pathway to arrive at endpoints of the task (e.g., using easier numbers or shapes). These PSTs were concerned that their open tasks might not provide productive struggles (NGSA, 2010) when students intentionally sought easier pathways and questioned the value of opening tasks in this way.

### Constraints with meeting standards and contexts

PSTs were asked to develop tasks that aligned with the chosen standards from Common Core State Standards for Mathematics (NGSA, 2010). Some PSTs shared their difficulties in developing open tasks when they were required to meet the specific curriculum standards in the assessment context. The following excerpts from PSTs' reflection highlight their uncertainties:

- "We were asked to address a specific standard. It was hard to make an open task because the standard is very specific about the problem setup."
- "Our goal [the goal/objective of our clinical interview] was to assess students' understanding of what they learned based on the standard. I am unsure how open the task can be used when assessing students."

Additionally, some PSTs revealed their beliefs that open tasks present greater challenges to students compared to closed tasks. For this reason, they also expressed that the brief one-on-one clinical interview setting posed a barrier to using more advanced levels of open tasks. The following excerpt highlights their perspective:

- "Usually, open problems are more difficult, and I did not want to make the problem too complicated when the time for the one-on-one interview is limited."

### Gaps in knowing and doing

PSTs took educational foundations courses before this mathematics methods course and learned about differentiated teaching. Thus, some PSTs stated that they wanted to incorporate the idea of differentiation into their development of open tasks. This could have involved utilizing tasks that could be approached at varying levels of difficulty, enabling all students to engage with the same task at a level appropriate for them. However, they felt a gap between what they knew and what they needed to do. One PST stated, "I thought I fully understood the idea of differentiation, but I realize that I don't know how to do it in mathematics, science, social studies, etc." This quote illustrates the disparity between understanding the concept of differentiated task design and the ability to effectively create a task that considers students' proficiency levels.

## Discussion and implications

This study addresses a gap in the literature. Although the ability to design and implement open mathematical tasks is a crucial skill for mathematics educators, little is known about the types of mathematical tasks that elementary PSTs consider as "open" and their abilities to create those tasks. Thus, this study examined elementary PSTs' preconceptions and development of open mathematical tasks, as well as their reflections on implementing them.

PSTs were asked to develop open tasks as part of their one-on-one clinical interviews with elementary students to assess their understanding and performance of topics they had already learned. Thus, the findings of this study are limited to this context. If PSTs were asked to develop long-term plans, they might have proposed different types or levels of open tasks. However, this study offers a snapshot of the PSTs' preconceptions and design of open tasks by zooming in on the characteristics of the tasks developed by them. This section revisits the findings from this study to compare them with prior research and to acknowledge remaining questions that may lead to further research and offer implications for teacher education.

## (Un) familiarity vs. consideration of contexts

It is worthwhile to note that the majority of PSTs (50 out of 56) proposed mathematical tasks with high cognitive demands. Of these, 47 proposed tasks involving procedures with connections, while three PSTs proposed tasks that required doing mathematics (Stein & Smith, 1988). Moreover, except for three completely closed tasks, other tasks have at least one open aspect when analyzed with categories of entry, method, and answer. This result suggests that PSTs enter a teacher education program with a certain degree of understanding of open mathematical tasks.

However, the types of tasks proposed by PSTs through task design are very limited. Although PSTs mentioned various characteristics of open mathematics tasks in the initial discussion, 95% of PSTs (53 out of 56 PSTs) proposed procedural tasks in the current study's context. Taking these results at face value, using different formats of tasks did not seem to be a useful strategy for PSTs to create open tasks. Klein and Leikin's (2020) study with in-service teachers also reported that participants were less familiar with investigative and sorting tasks and less frequently posed such task formats. Thus, this result might be due to PSTs' unfamiliarity with using and creating a variety of open task formats. At the same time, some PSTs' post-reflections implied that the context of the short one-on-one clinical interview might have prevented them from utilizing various task formats. In other words, PSTs tended not to use atypical formats of tasks open too widely when they were obligated to address specific standards or learning objectives and make an evaluative judgment regarding students' performance. Also, during the reality check (i.e., working with elementary students), some PSTs noticed that their students were not familiar with open tasks, indicating that such tasks are not commonly employed in real classroom settings. Consequently, these PSTs seemed to step back from appreciating the value of open tasks.

Considering PST's competencies in some aspects, mathematics teacher education programs need to provide more guidance and learning opportunities to PSTs regarding open task development. Teacher educators also need to recognize the tacit knowledge and intuition of PSTs (Markauskite & Goodyear, 2014) that they bring to the coursework and utilize them as pedagogical resources. For example, 10 tasks were coded as closed methods tasks as PSTs did not explicitly state to solve the task in multiple ways. Thus, teacher educators may instruct PSTs to clearly state the task statements to align the intention of task designers and the interpretation of problem solvers.

### Openness: PSTs' preference and remaining wonderings

As a task can be open in various aspects, the openness of PSTs' tasks was examined focusing on several components: entry, method, and answer. It is worth revisiting a few observations and remaining wonders to unpack their meaning and guide further study.

#### Scope of openness

In terms of goal, all PSTs' tasks were identified as closed because all contain a specific goal in the task statement. Thus, opening the goal did not seem to be PSTs' preference. As discussed in the previous section, this might reflect PSTs' preference of not too widely opening the task, especially in the context of clinical interviews with elementary students. Similarly, PSTs showed a lack of variety in complexity and extension of the open tasks.

For other components (entry, method, and answer), PSTs more actively attempted to open the task. Like Klein and Leikin's (2020) study with in-service teachers, multiple strategies and outcomes tasks were the common types of open tasks posed by the PSTs in this study. Additionally, it is worth noting that the participants in this study regarded opening entry as an equally viable approach, as evidenced by their justifications and work samples (see Tables 5 and 6). Even when the PSTs had a closed-entry case ($n=26$), most of them ($n=23$) had multiple ways of solving the task (open method), which shows PSTs' ability to create an open task to a certain degree. However, it was rare to see cases that had multiple open components. For example, when the entry was open, a substantial number of tasks were identified as closed method tasks, whereas when the entry was closed, PSTs tended to open methods (see Table 6).

The remaining question is what the optimal level of openness PSTs need to reach for the development and enactment of meaningful tasks during their training at a teacher education program. We do not believe that there is one correct answer regarding the optimal level of task openness. Instead, teachers need to have competencies in designing open tasks considering the characteristics of learning topics. For example, when learning challenging and new topics, teachers might open only one or two aspects of tasks (e.g., entry, method, and goal). Conversely, when learning easy and familiar topics, teachers can open all aspects of tasks. Moreover, teachers might need to adjust task openness considering the mathematical abilities of their students to provide differentiated instruction. To achieve these goals, PSTs should understand various aspects of open tasks and have competencies in designing open tasks.

A subsequent question lies in what types of support and experiences mathematics teacher education programs need to provide to PSTs. We believe that the theoretical framework used in this paper can be a valuable pedagogical resource to guide PSTs' knowledge and skills in developing open tasks. For example, mathematics teacher educators may ask PSTs to conceptualize, develop, and reflect on open tasks. Then, PSTs could analyze their products with their peers and teacher educators using the several components presented in this study (e.g., goal, method, complexity, answer, and extension). These stakeholders can collaboratively provide feedback to one another to help PSTs acquire critical knowledge, resources, and skills to understand and execute open tasks.

### Difficulty in maintaining cognitive challenges in open entry tasks

Over half of PSTs designed open entry tasks (54%) in which partner elementary students were asked to pose problems. Prior research has not focused much on open entry tasks, which was one of this study's novel aspects. However, we noticed a couple of patterns in PSTs' work samples. First, PSTs' task development reports were less sophisticated for the open entry tasks because they were unsure about what problems the students would come up with. Second, some PSTs found that students did not always choose to tackle the more challenging problems when given the freedom to pick their tasks. This was contrary to the PSTs' expectation that opening up the task options would lead to more challenging choices. As a result, the PSTs felt that they were unable to maintain the desired level of challenges during one-on-one clinical interviews. PSTs may have noticed that simply having open task features (such as open entry) does not guarantee the desired cognitive challenges. Similarly, cognitively challenging tasks may not always be in the form of open tasks.

The remaining question in teacher education is how we can support PSTs in developing and using open tasks while maintaining the cognitive challenges of the tasks. Teacher educators may instruct PSTs in teaching strategies to maintain cognitive challenges during task implementation, such as using instructional discourse, fostering student engagement, and encouraging student self-monitoring (Hong & Choi, 2019).

### Intended openness vs. assumed openness

Prior studies with in-service teachers (e.g., Klein & Leiken, 2020) report that multiple solution tasks were the most used and familiar type of open task for teachers. Findings from PSTs in this study are similar to prior studies in their forms, but there are several unique aspects found in our research setting that are worth revisiting.

About 57% of PSTs' tasks were categorized as closed. However, it might not be the complete picture of PSTs' intention. These tasks mostly took the form of problems used in number talks (Parrish, 2011), which typically present a mental mathematics problem and ask students to use various strategies to solve it using number relationships and the structures of numbers. When looking into PSTs' proposed answers, they indeed presented solutions with multiple approaches. Because planning and implementing number talks were part of the course, it was not a surprise. This result implies that the types of open mathematical tasks that PSTs develop are related to the content that they are exposed to in mathematics teacher education courses.

Also, most mathematical problems can indeed be solved with different strategies or solution methods. However, due to the absence of explicit reference in the task instruction for using multiple methods, these tasks were categorized into closed method tasks, following justification from a prior study (Bingölbali & Bingölbali, 2020). In other words, the PSTs assumed openness in solution methods, but their intention regarding openness was not explicitly presented in the task context.

One positive takeaway from these findings is that most PSTs do not believe there is only one correct way to solve mathematical problems (Lewis, 2005; Schoenfeld, 1992). The remaining question is how the work of designing and executing open tasks can be productively used in teacher education and what potential work has in enriching teacher education. We suggest that mathematics teacher educators and researchers need to recognize PSTs' preconceptions and competencies in task development and provide support to elevate their expertise. As Superfine (2021) claimed, "such an asset-based perspective positions PSTs as pedagogically competent in contrast to reinforcing the knowledge and skills that are likely underdeveloped or even lacking" (p. 331). For example, teacher educators can ask PSTs to use explicit references when designing open tasks (e.g., "Find the total number of pencils in three glasses using different strategies").

Further studies can take the form of a design-based inquiry aimed at developing mathematics teacher education courses and sessions dedicated to enhancing the knowledge and skills of PSTs in this area. These studies can also investigate the adequate tools and environments for achieving these objectives. For example, as noted at the beginning of the discussion section, our approach was confined to a single clinical interview. Future studies could explore how engaging in multiple clinical interviews might enhance PSTs' understanding of open tasks and their ability to implement these tasks as intended. Based on a previous study highlighted the positive relationship between PSTs' clinical interview skills and teacher noticing (Lee, 2021), it is plausible that as PSTs develop their clinical interview skills over time, their abilities to design and enact open tasks may also improve.

## Concluding remarks

This study did not intend to claim that using open tasks is the only way to teach and learn mathematics; instead, it is a part of the broad spectrum of various options teachers can consider to teach and learn mathematics meaningfully. Considering this potential, this study's findings showed that the PSTs' preconceptions of open tasks address various aspects of open tasks identified in the literature. However, a much narrower spectrum of aspects was incorporated when designing open tasks and implementing them with elementary students, despite their ability to create an open task to a certain degree. As other studies on different topics report, this may indicate the gap between what PSTs think and do (e.g., Lee, 2012). Also, this may imply that PSTs' anticipated challenges might weigh more than their expected benefits of open mathematical tasks. Thus, the empirical studies regarding PSTs' perceived benefits and anticipated challenges continue to be a desideratum.

While this study's scope mainly focused on the breadth of PSTs' preconceptions and development of open tasks, the depth of the task quality may be further probed in subsequent studies. Instead of using open tasks for open task's sake in their form, teacher educators need to provide more support for PSTs to implement open tasks to promote mathematical reasoning and understanding among students. As PSTs demonstrated their competency in defining open tasks and associated characteristics, more efforts should be placed on refining their use of open tasks with intentionality. Additionally, there is a need to expand the variety of open tasks that PSTs design by refining their knowledge and skills in teacher education programs. This should also include considerations for teaching mathematics to elementary students.

## Declarations

**Conflict of interest** On behalf of all authors, the corresponding author states that there is no conflict of interest.

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