J. Korean Soc. Math. Ed. Ser. D. (2024) 27(1), 25–47
https://doi.org/10.7468/jksmed.2024.27.1.25

ISSN 1226-6191
Online ISSN 2287-9943

RESEARCH ARTICLE

U.S. and Korean teacher candidates' approaches to mathematical modeling on a social justice issue

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Received: January 8, 2024 / Revised: January 31, 2024 / Accepted: January 31, 2024

Abstract

Mathematical modeling activities are gaining popularity in K-12 mathematics education curricula worldwide. These activities serve dual purposes by aiding students in making sense of real-world situations intertwined with social justice while acquiring mathematical knowledge. Despite efforts to prepare teacher candidates for instructing in mathematical modeling within a single country, little attention has been given to teacher candidates' approaches to mathematical modeling on a social justice issue from different countries. This article employs an in-depth, small-scale comparative study to examine the approaches of U.S. and Korean teacher candidates in solving a justice-oriented mathematics task. Our findings reveal that, although both U.S. and Korean teacher candidates identified certain variables as key when constructing a mathematical model, Korean teacher candidates formulated a more nuanced model than U.S. candidates by considering diverse variables. However, U.S. teacher candidates exhibited a heightened engagement in linking the task to social justice issues, whereas Korean teacher candidates barely perceived real-world problems in relation to social justice concerns. This study serves as a valuable tool to inform the roles and limitations of teacher education programs, shaped within specific educational contexts.

Keywords: mathematical modeling, social justice, mathematical task, preservice teachers, comparative study

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I. INTRODUCTION

Mathematical modeling is an integral part of mathematics curricula (e.g., Alhammouri & Dinapoli, 2023; National Governors Association Center for Best Practices and the Council of Chief State School Officers [NGA Center for Best Practices], 2010; HanKeln, 2020; Ministry of Education [MOE], 2019). Researchers have studied mathematical modeling to teach mathematical knowledge and increase students' problem-solving abilities (Niss et al., 2007). Some researchers have broadened the area of mathematical modeling and incorporated it in mathematics education from a socio-critical perspective (Abassian et al., 2019). They have explored how students engage in mathematical modeling when its task involves social justice issues. This approach aligns with scholars advocating for social justice in mathematics classrooms, which encourages students to "investigate and critique events, situations, and actions of public interest, and to challenge oppressive structures and acts that prevent people from reading and writing the world" (Frankenstein, 1990; Freire, 1970; Martin & McGee, 2009, cited by Aguirre et al, 2019, p. 7). This perspective regards justice-oriented mathematical modeling as a tool to improve cultural responsiveness in learning environments. This perspective also aims to enhance students' competency in mathematical modeling while increasing awareness of critical consciousness on societal issues. (Aguirre et al., 2019; Barbosa, 2006; Jung & Magiera, 2023).

As mathematical modeling problems are connected to real-life contexts, they can be interpreted in different ways depending on multiple perspectives (Maaß, 2006). Students need to understand the context of a problem, select certain variables, and discard other variables to construct a model (Lesh et al., 2008; Jung & Brady, 2023). The processes of understanding, constructing, and interpreting process are intertwined with students' knowledge, experiences, perspectives, and classroom discourses (Barbosa, 2009). Thus, researchers may gain insights into students' sociocultural background and their perspectives on social justice using mathematical modeling activities.

Despite growing attention that teaching mathematical modeling in connection to social-justice issues, most studies were implemented within a single country (Hankeln, 2020). Thus, there were few studies that compared the mathematical modeling processes of students from different countries, drawing on a socio-critical perspective. Particularly noteworthy is the limited attention paid to preservice teachers (PSTs), despite their need to acquire mathematical knowledge and teaching skills during college students, shaping them into future mathematics educators.

Recognizing that "in-depth, small-scale international comparative studies can provide unique opportunities for us to understand students' mathematical thinking" (Cai et al., 2017, p. 96), this study aims to shed light on diverse mathematics thinking and rationales of the PSTs from different countries during the mathematical modeling process. We selected PSTs from the U.S. and Korea as examples. Despite the unique teaching and teaching traditions in each country (Association of Mathematics Teacher Educators [AMTE], 2017; Park, 2016), these countries use each other as reference points to contemplate their teacher education. Korean teacher educators look to U.S. teacher education for inspiration to enhance their own teacher training programs while U.S. teacher educators frequently commend high teacher quality (Kim et al., 2011).

The findings of this study can provide an opportunity to understand both the converging and diverging perspectives of PSTs on mathematical modeling from each country. This insight serves as a valuable means to reflect on the current roles and limitations of teacher education programs, shaped within specific educational contexts. It can inform important directions in shaping the epistemic models of mathematics teacher education on what aspects of mathematics processes and practices need to be addressed. The following research question guided the inquiry of this study: What similarities and differences do U.S. and Korean PSTs show when solving justice-oriented mathematics task?

II. LITERATURE REVIEW

Mathematical Modeling

Mathematical modeling is "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision" (NGA Center for Best Practices, 2010, p. 72). Teachers can achieve two goals – fostering students' understanding of real contexts and mathematical abilities – by incorporating mathematical modeling activities (Bardy & Fehlmann, 2023; Han & Hwang, 2023; Julie & Mudaly, 2007; Jung & Brady, 2023; Schukajlow et al., 2018). As "mathematical modelling always originates from a real-life problem, which is then described by a mathematical model and solved [the problem] using this model" (Greefrath & Vorhölter, 2016, p. 8), students can learn how to make sense of the context of the problem, collect data, construct a model, and interpret solutions to solve a task. Thus, students can learn how to understand and interpret real contexts through a mathematical modeling activity.

Furthermore, mathematical modeling tasks can enhance learning motivation of students to engage in mathematics (Zbiek & Conner, 2006). As modeling serves as "a vehicle for facilitation and support of students' learning of mathematics as a subject" (Niss et al., 2007, p. 5), teachers can use a mathematical modeling task as an example to introduce and teach mathematical knowledge (Abassian et al., 2019). Students can apply acquired mathematical knowledge to understand reality and acquire new mathematical knowledge by solving a mathematical modeling task (Schukajlow et al., 2018; Hankeln, 2020)

Mathematical modeling is characterized by iterative sequential stages, albeit with slight variations in how researchers articulate these stages (Anhalt & Cortez, 2015; Aguirre et al., 2019; Suh et al., 2021). Generally, it entails six stages: (1) making sense of real-world situations, (2) devising a model through assumptions and data collection, (3) computing the solution of the model, (4) interpreting the solution and drawing conclusions, and (5) validating these conclusions. Then, students report their solutions along with the rationale behind them. Figure 1 shows these modeling process consisted of simplifying, mathematizing, working with mathematics, interpreting, and validating (Maaß, 2006).

While all stages are important, simplifying the given real-world situations in mathematical terms, a process known as 'mathematizing' is critical. Maaß (2006) explained it as "By simplifying, structuring, and idealizing this problem [a real-world problem,] you get a real model (p. 115)." These processes require students to choose variables and develop a model based on their understanding of the real context, enabling them to apply, examine, and broaden their acquired mathematical knowledge. Additionally, by emphasizing mathematizing processes, teachers can assess students' modeling competency and their perspectives of a society by analyzing the variables they consider as important aspects for solving the task.

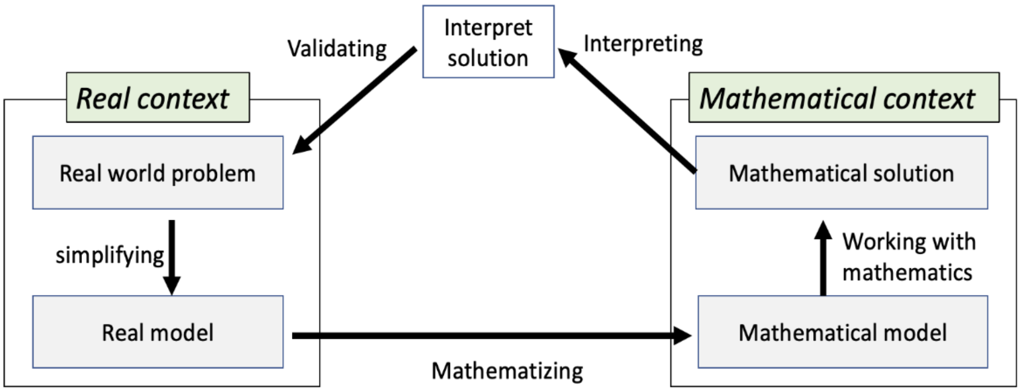


Figure 1. Simplified mathematical modeling process (Adopted from Maaß, 2006)

Socio Critical Mathematical Modeling

Researchers have emphasized the role of mathematics in society as mathematics influences people's decision-making process (Barbosa, 2006). People understand and critique society. They also consider certain variables more important than others in the decision-making process, sparkling discussions on whether the outcome is fair or contributes to enhance social justice. In this context, the emphasis on equity and justice in mathematics education has grown (Jung & Magiera, 2023; White et al., 2016).

National Council of Teachers of Mathematics [NCTM] has consistently promoted equity in mathematics education programs, asserting that regardless of students' backgrounds, all students should be provided a high-quality mathematics teaching and sufficient support and resources to maximize their potential (NCTM, 2014, 2020). Scholars have also argued that social justice issues should be integrated in mathematics teacher education programs as they help PSTs acquire relevant knowledge and prepare them to address those concerns in their future classrooms (White et al., 2016). AMTE (2017) advocates that "equity must be both addressed in its own right and embedded within every standard for preparing mathematics teachers" (p. 1). To achieve these goals, some mathematics teacher educators have used justice-oriented mathematical modeling tasks to help PSTs think about realistic problems based on the social critical perspectives (Felton-Koestler, 2020; NCTM, 2020).

The mathematical modeling process is not neutral. The model construction process reflects not only how students see and understand the real world situations of a mathematical problem but also how their lived experiences and multiple knowledge bases affect the ways that they create a mathematical model (Suh et al., 2021). Therefore, solving justice-oriented mathematics task empowers learners and allow them to reflect their perspectives in/towards society (Felton-Koestler, 2020). PSTs have “opportunities to understand and challenge inequitable situations and examine the distribution of resources, privileges, and power among cultural, socioeconomic, and environmental groups” (Jung & Magiera, 2023, p. 233). In this perspective, scholars have examined how students, PSTs, and teachers solve justice-oriented mathematical modeling tasks.

For example, Aguirre et al. (2019) presented teachers with The Flint Water task (see Table 1) and asked them to figure out how much water will be needed for students in Flint. To solve this task, teachers need to decide several variables, such as the number of students in Flint, the water needed for individual students per day, the remaining days in 2016, and the size of bottles. In addition, some teachers considered the age of students, such as from 5 to 8, from 9 to 12, and over 12 to solve the task. They view that the equal water distribution based on a concept of fair distribution is not a reasonable method as the older students need the more water from the equity perspective. Thus, the teachers believe that students need to be provided different amount of resources according to individual students’ needs to achieve the similar outcomes.

Table 1. Simplified version of Flint Water task (adopted from Aguirre et al., 2019)

Context: The water in Flint, Michigan was contaminated due to lead. Several companies decided to donate water for over 10,000 schoolchildren for the rest of the calendar year.
Question: “How much water will be enough to meet the daily needs of Flint schoolchildren until December 31, 2016” (Aguirre et al., 2019, p. 9)

Similarly, Casey et al. (2023) asked PSTs to engaging in a statistical modeling task where they distribute school fundings across the state. Some PSTs responded that equal distribution is not a reasonable method, and funding should be allocated by considering demographic information for each school district (e.g., students below the poverty line). Using the problem-posing, Jung and Magiera (2023) asked PSTs to pose mathematical modeling problems addressing social justice issues. Although PSTs had learned how to pose a problem and engage in mathematical modeling activities addressing social justice issues, the mathematical problems that they had created did not include some features needed for mathematical modeling, including model development and justification. Thus, Jung and Magiera emphasized that when addressing a justice-oriented mathematical modeling task for PSTs, emphasis should be placed on mathematical model and justification processes. However, current literature provides limited information regarding the differences among PSTs from various backgrounds in their engagement in a justice-oriented mathematics task.

Mathematical modeling in Korean and U.S. mathematics education

In the U.S., mathematical modeling has gained increased interest from researchers and educators, especially with the inclusion of Standard for Mathematical Practice 4 ('Model with Mathematics') in the Common Core State Standards for Mathematics [CCSS-M] (NGA Center for Best Practices, 2010). Mathematics teachers acknowledge that mathematical modeling can enhance equitable mathematics instruction by providing students with various entry points to grasp mathematical content (Fulton et al., 2019) and nurturing their agency in the process of doing mathematical modeling (NCTM, 2020). AMTE encourages U.S. mathematics teacher education programs to equip PSTs with a robust and flexible knowledge of mathematical processes and practices. Specifically, *the AMTE Standards for Preparing Teachers of Mathematics* (2017), as demonstrated in their standards C.1.2., emphasizes the ability of well-prepared beginning teachers of mathematics as the following: "They (well-prepared beginning teachers of mathematics) can apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and the students they will teach (p. 9)." However, most U.S. teachers have had limited exposure to mathematical modeling (Anhalt & Cortez, 2015). Hence, many U.S. students and PSTs do not have sufficient experiences construing a mathematical model (Felton-Koestler, 2020; Jung & Magiera, 2023).

The Korean mathematics curriculum do not emphasize mathematical modeling as much as the U.S. CCSS-M (NGA Center for Best Practices, 2010). In the South Korean 2022 revised mathematics curriculum, mathematical modeling was introduced as one of teaching and learning strategies. However, the curriculum did not explain what mathematical modeling process is and how it should be taught to students (South Korean Ministry of Education [MOE], 2022). Similarly, the guide book of the 2015 curriculum for mathematics teachers that were distributed by MOE to supplement a mathematics textbook simply introduced mathematical modeling as one of strategies to teach problem-solving competency (MOE, 2019). Thus, teachers and PSTs are barely aware of the importance and process of mathematical modeling, leading to most students also having very limited experiences in learning mathematic with modeling (Kim, 2022).

Conceptual Framework: Justice-oriented Mathematical Modeling

Social justice is a social and ideological process (Gates & Jorgensen, 2009). Individuals vary in the extent to which they believe in social justice and mathematical knowledge, leading to divergence in the mathematical modeling process (Casey et al., 2023; Jung & Magiera, 2023). We adopted a conceptual framework proposed by Aguirre et al. (2019) which connects social justice and mathematical modeling (see Figure 2); the left cycle related to selecting social justice issues (broad social issue, real-world situation, social awareness & sense of action), and the right cycle related to solving the task using mathematical modeling.

Based on the conceptual framework, this paper studies how PSTs solve a justice-oriented mathematical modeling task. We attend to how PSTs make sense of the task, formulate a mathematical model, and share the mathematical model. We also examine how PSTs engage in both a mathematical modeling cycle and a social justice cycle. Moreover, we analyze how PSTs address social justice issues (Aguirre et al., 2019; Gates & Jorgensen, 2009). We ask whether PSTs focus on fairness in access to resources or interrogating social justice. Although the former (‘fairness’) addresses the importance of resource distribution, it presumes the continuation of status quo. However, the latter (social justice) problematizes unjust practices and barriers by promoting systematic changes to resolve structural inequality. We intended this approach to reveal how U.S. and Korean PSTs approach a justice-oriented mathematics task.

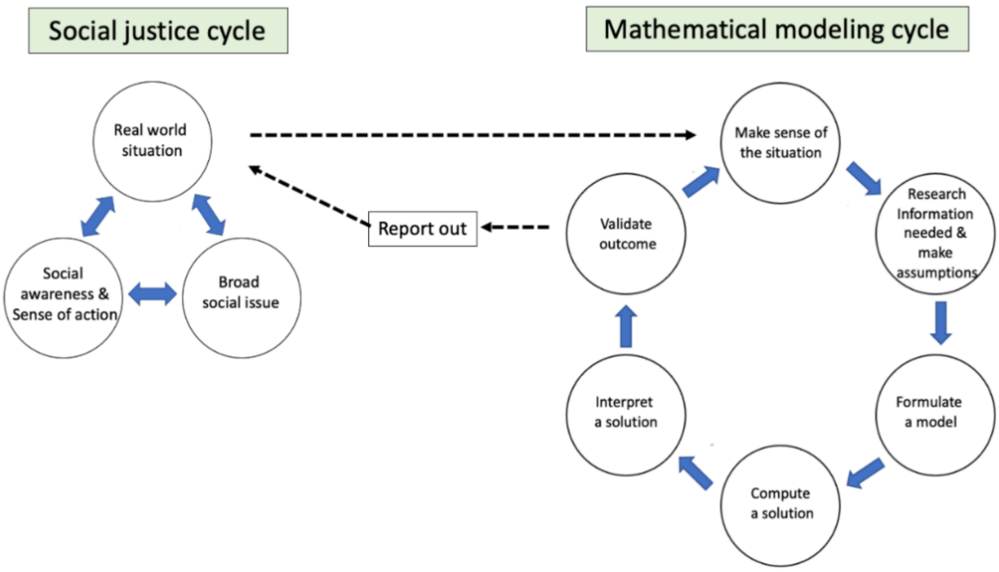


Figure 2. A conceptual framework of this study (Adopted from Aguirre et al., 2019)

III. METHODS

Participants and Context

The participants in this study consisted of seventeen PSTs in the U.S. and twenty-five PSTs in Korea. They were all enrolled in a mathematics method course in a teacher education program at a university. All participants were self-selected volunteers who signed an IRB-approved consent form. The study involved a justice-oriented mathematical modeling task with the prompt focused on the Flint Water task (Aguirre et al., 2019). This task was conducted during regular class time. As the authors of the paper were the instructors of the courses, we acknowledged the power dynamics between us and the research participants. Thus, we minimized the possibility of undue influence by informing participants that their involvement in the study would not be revealed to the course instructor until after the course grading was completed. During the course, we provided an overview of the mathematical modeling process and asked PSTs to solve a practice task (The Elevator task, see Table 2) before the research began.

Table 2. The Elevator task (Adopted from Kang, 2020)

Context: At 9 a.m., Dave entered the class after the bell rang, marking the fourth time he had been late this month. He felt it was unfair because he had arrived at school on time, but he ended up being late due to the crowd waiting for the elevator. Today, 60 students arrived in the classroom slightly after 9 o'clock. The school has six floors. However, due to budget constraints, installing more elevators or increasing the capacity of one elevator is not possible.
Dave and his friends began to contemplate whether there was an effective way to use the elevator.
Question: How should we operate elevators during morning school hours?

Task Development and Implementation

To analyze PSTs' mathematical modeling process regarding just-oriented perspectives, we adapted the Flint Water task developed by Aguirre et al. (2019). We selected the task as it addressed social justice issues and U.S. and Korean PSTs were familiar with the water pollution issues. In U.S. the Flint water crisis in Michigan was widely reported in 2016 and U.S. PSTs noticed the dangers of lead-contaminated water in Flint. Similarly, Korean news reported that heavy metals (e.g., lead, arsenic, and copper in the river) were widely spread in the city, Sokcho county, where the Korean PSTs were acquainted with it (Kim, 2020). Thus, the Flint Water task could evoke the respective PSTs' personal and collective interests to engage in justice-oriented mathematical modeling.

The original Flint Water task (Aguirre et al., 2019) was modified in three aspects. First, we included one more variable (protein bars) to enrich the discussion of PSTs. Second, we provided information about the number of schoolchildren and the start day of the water distribution (January 1st) to compare PSTs' solutions because if this information was different across groups, their solutions would not be comparable. Third, we provided explanation for both U.S. PSTs and Korean PSTs that water pollution can disproportionately harm marginalized populations. This explanation was delivered to help PSTs seriously consider the repercussions of this water pollution issue. We did not provide further information on the task and its context (see Table 3); instead, we asked PSTs to find information and make assumptions (e.g., the size of the bottle) to solve the task following the Aguirre et al. (2019)'s study. In the Korean version task, we changed the name of the city Flint to Sokcho.

The U.S. and Korean instructors created a protocol following the three steps for comparative analysis of the data (Table 4). Since U.S and Korean instructors co-designed the task implementation process and followed the protocol, their instructional practices of three phases were similar. Each group, consisting of two to five PSTs, participated in this task by undertaking these phases. The instructors set the norms before the group activity to ensure that PSTs actively engage in mathematical modeling and respect one another's approaches in solving the task. Both U.S. and Korean PSTs actively engaged in the group activity.

During the first phase ('making sense of the task'), both instructors at each research site introduced the modified Flint Water task through images of the water crisis. The U.S. instructor used a couple of photos for the Flint water crisis while the Korean instructor used other images taken in Sokcho county in Kwangwon providence, Korea. The images showed lead-tainted water bottles, water bottles being donated to local residents, and children drinking from water bottles. After the rich conversation based on photos (e.g., what, why, and how this crisis occurs), the instructor presented a prompt explaining the company's plan and the related question (see Table 3).

Table 3. Modified Flint Water task used in this study (Adopted from Aguirre et al., 2019)

The Flint Water task (distributed to U.S. PST)
Context: Due to lead-contaminated water, school children in Flint, Michigan are experiencing a food and water shortage. Flint has a lower median income compared to its neighboring cities. On January 1st, Walmart announced that they are planning to donate boxes of protein bars and water to meet the daily needs of 10,000 schoolchildren for the remainder of the calendar year. Question: What quantity of protein bars and water would be sufficient to feed school children in Flint throughout the school year?
The Sokcho Water task (distributed to Korean PST)
(Translation – Context: 납으로 오염된 물로 인해 속초시 학생들은 식량과 물 부족을 겪고 있습니다. 속초시는 인근 도시에 비해 주민들의 월 평균 소득이 낮습니다. 이마트는 1 월 1 일부터 1 년동안 속초시에 살고 있는 10,000 명의 학생들에게 매일 단백질바와 페트병 물을 기부할 계획이라고 발표했습니다.) (Translation – Question: 속초시의 학생들이 일년 동안 먹기에 충분한 양의 단백질바와 물은 어느 정도인지 구하시오)

During the second phase ('formulating a mathematical model'), each group made assumptions, identified key variables related to the task, gathered information using their own laptops, and created a mathematics model (about 40 minutes). To solve the task, PSTs gathered information for the variables that they identified relevant based on their assumptions. This process led them to construct a mathematical model. Then, they calculated what quantity of protein bars and water would be sufficient to feed school children in Flint/Sokcho throughout the school year.

During the third phase ('sharing the models'), each group explained and justified their mathematical models and outcomes, and other groups questioned and criticized them (about 30 minutes). Each group of PSTs wrote down their modeling process on a whiteboard, which was hung on the wall of the classroom. They showed their assumptions, key variables and quantities, the developed models (e.g., equations, descriptions), their interpretation, conclusions, and validations. Each group explained their modeling processes and described why they made specific assumptions and how their assumptions impacted their models and outcomes. After each group's presentation, the floor was open for everyone to reflect on the similarities and differences and share any thoughts they wanted to express.

Table 4. Process of justice-oriented mathematical modeling task implementation

The first phase (10 min) **Making sense of the task**	The second phase (40 min) **Formulating a mathematical model**	The third phase (30 min) **Sharing the models**
An instructor introduces the task, which is situated in a real-life scenario to help PSTs make sense of it. As an open-ended task, the given problem has multiple solution paths for approaching it.	PSTs research and select variables relevant to the task based on their assumptions. PSTs document and elicit mathematical modeling using symbolic, numerical, tabular, or graphical expression. This process can be reiterative, allowing students to elicit, modify, and improve the model.	PSTs share their mathematical models, processes, and solutions with their colleagues and instructor. PSTs understand and criticize mathematical models of one another. This process is designed to evoke justifications for mathematical models and outcomes beyond sharing single numerical answers.

Data Sources

Three types of data were used. First, field notes were taken by each researcher, describing the observed conversations within and between groups. The notes also include how the PSTs responded to the task. Second, hard copies of the task were collected, where each group described their thoughts and mathematical modeling process. These hard copies showed the variables that PSTs found relevant to the task and information they found from internet searching (e.g., the recommended daily protein intake). Third, the instructors used their mobile phone to audio record group discussions of each class on the modified Flint Water task.

Data Analysis

We analyzed the collected data by focusing on two cycles (mathematical modeling and social justice cycles, see Figure 2), which is also aligned with Table 4, to make sense of how PSTs engage in a mathematical modeling process when solving a justice-oriented mathematics task. First, regarding the mathematical modeling cycle, we examined how PSTs formulated mathematical models (the second phase of task implementation), and how PSTs explain and justify their models and conclusions (the third phase of task implementation). Table 5 shows the samples questions used for data analysis.

Second, regarding the social justice cycle, we interrogated which forms of equity are shown in the approaches taken by the PSTs (Gates & Jorgensen, 2009). Specifically, we looked into whether PSTs prioritize a stance focusing on equal distribution of resources, which we named as the 'fairness stance' or whether they attend to equal outcomes and social structural problems in society beyond fair distribution of resources, which we named as the 'social justice stance'. Subsequently, we analyzed the similarities and differences between U.S. and Korean PSTs in their conceptualization of equity in engaging in the given modeling task.

Table 5. Data analysis process focusing on mathematical modeling

Analysis for the second phase (**Formulating a mathematical model**)	Analysis for the third phase (**Sharing the models**)
- What variables that PSTs attend to - What information relevant to the variables PSTs collect - What assumptions PSTs make - What mathematical model PSTs create - How PSTs compute solutions	- How PSTs interpret solutions - How PSTs validate outcomes

IV. FINDINGS

This section describes the mathematical modeling process undertaken by the PSTs for the modified Flint Water task from the U.S. and Korea. We discussed the mathematical modeling processes of U.S and Korean PSTs, uncovering both the similarities and differences in their approaches when tackling a justice-oriented mathematical modeling task.

Mathematical Modeling Process of U.S. PSTs

Selected Variables to Formulate a Mathematical Model. To construct a model to answer the question ("What quantity of protein bars and water would be sufficient to feed school children in Flint throughout the school year?"), most U.S. PSTs constructed formulas including (*the number of school days* × *the number of school children* × *the number of protein bars needed per day*) + (*the number of school days* × *the number of school children* × *the number of water bottles needed per day*). Thus, as we provided the information of the number of school children (10,000), their investigation usually focused on finding *the number of school days, the number of protein bars needed per day,* and *the number of water bottles needed per day* (see Table 6).

First, regarding *the number of school days*, PSTs interpreted the number of school days in three different ways (110 days, 180 days, and 365 days), while the task explained that Walmart will distribute the resources from January 1st. Two groups (Group 2 and Group 3) identified it as 180 days considering the general school days in U.S., whereas the other two groups (which groups) chose 365 days based on the calendar days. One group (Group 1) identified it as 110 days (180 days 70 days), counting from the start of the Fall semester to the day they solved the task by ignoring the information provided in the prompt. However, one group (Group 6) didn't consider school days to formulate a model.

Second, regarding *the number of protein bars needed per day,* U.S. PSTs focused on (a) how much protein schoolchildren need per day and (b) the amount of protein in a protein bar. For example, Group 1 and Group 3 identified about 20 grams of protein as the requirement for schoolchildren set by the U.S. Department of Health. They also discovered that a protein bar contains 10 grams of protein, leading them to provide 2 protein bars per student (10 grams of protein in a protein bar × 2 protein bars = 20 grams of protein). In contrast, Group 6 delved into more detailed information by examining the amount of protein needed by age bracket (e.g., elementary, middle, and high school students). However, they could not calculate the specific numbers of protein bars.

Group 4 focused on the total calories in a day, not protein. They concluded to provide children with 15 protein bars in a day because the typical protein bars have 80 kcal, and school children need to intake 2000 kcal in a day. The remaining two groups (Group 2 and Group 5) did not look up how much protein school children need a day; instead, they assumed the quantities based on the daily routines of people's eating habits. Thus, they proposed to provide three protein bars for breakfast, lunch, and dinner.

Third, regarding *the number of water bottles needed per day,* U.S. PSTs focused on (a) how much amount of water schoolchildren need per day and (b) the amount of water in a bottle. Using information on a portal website (i.e., *Google*), five groups identified that school children need to intake 76 ounces per day (Group 1), 64 ounces per day (Group 2 and Group 3), and 80 ounces per day (Group 4). These differences might result from the types of databases they used. For example, Groups 2 and 3 gathered this information by typing 'how much water should children need to drink?' on Google and decided to provide four bottles of 16 ounces water (64 ounces water per day ÷ 16 ounces water in a bottle = four bottles of water).

Group 4 suggested to provide 80 ounces per day. They assumed that students needed water for drinking and washing their faces because the Flint water crisis generated lead water contamination. Thus, Group 4 proposed to provide one more bottle of 16 ounces. However, Group 5 did not look up the required water amount online but relied solely on their knowledge on their daily routine and chose five water bottles, saying, "Three to wash down protein bars, one when you wake up, one before going to sleep" (Group 5).

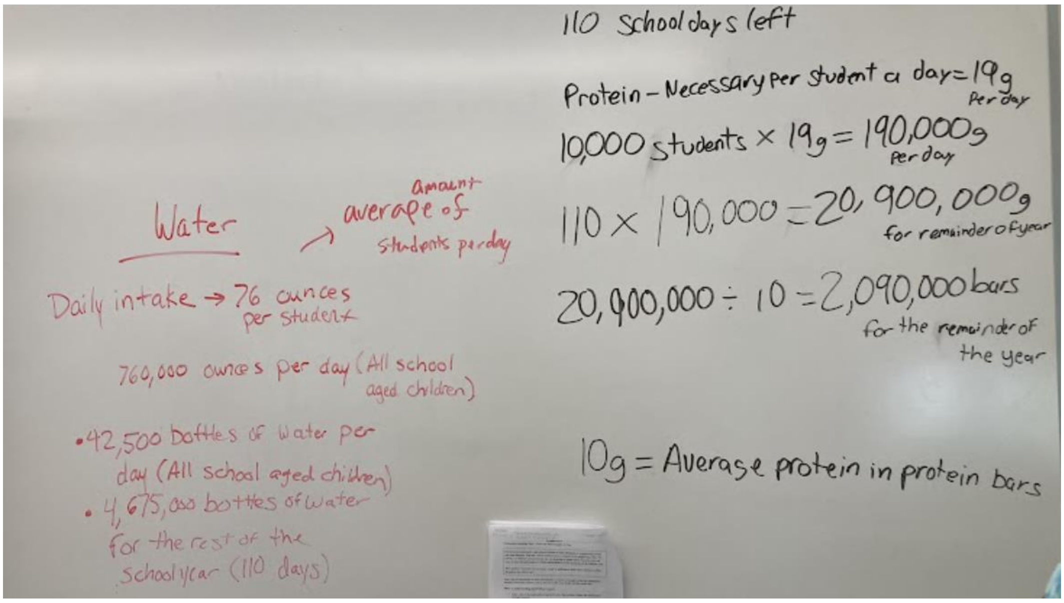


Figure 3. The mathematical modeling process of U.S. Group 1

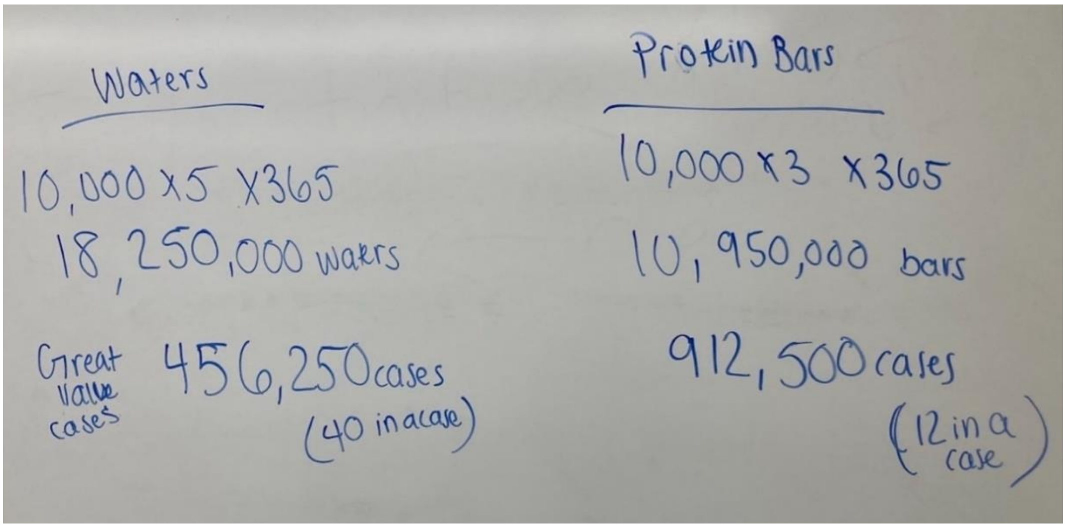


Figure 4. The mathematical modeling process of U.S. Group 5

Other Considered Variables and the Stance of Social Justice. Although most variables needed to construct a model were discussed throughout the first and second phases, PSTs explicitly and collectively shared health issues during the third phase. All six groups of the U.S. PSTs concerned about the health of students in Flint. For example, Group 3 was concerned about potential diseases that school children in Flint could face due to malnutrition as well as overconsumption of protein, stating “It is unsafe (to give only protein bars.) If you consume only protein calories, you will develop something called ketoacidosis, which poisons your blood.” Group 5 groups also considered lead contamination in Flint when determining the required water quantity, while they did not use it to formulate a model. Thus, they suggested providing more water needed for taking a shower.

Regarding the social justice cycle, all groups identified the Flint Water task as a social justice issue. They asserted that they need to ensure that all the students receive the amounts of water and protein bars as they needed, rather than distributing additional amount of these resources. Their stances went beyond the fairness stance (Gates & Jorgensen, 2009). Group 1 and Group 2 aimed to provide more resources beyond the minimum daily intake requirement for students. Group 1 stated that “If you're going to consider this for equity, you need to add more water and protein bars. Because you don't know who needs more. …We have that extra for the unknown circumstances.” They recognized that as students in Flint were affected by water contamination, they should be provided more resources beyond the daily water and protein intake requirement for ordinary situations. They argued for supplementary resource distribution that are catered to differential needs to achieve equal health status similar to ordinary students.

The other four groups (G3, G4, G5, and G6) pinpointed to the low SES of the city of Flint to discuss why social justice-oriented approach is needed in this mathematical modeling process (Gates & Jorgensen, 2009). For example, Group 3 shared empathy towards status quo in Flint where a large population had already suffered from limited resources that perpetuated low SES, “Flint, Michigan is an ongoing issue. It is not something that just happened and everyone forgets. They are not having a problem just with water and contamination.” She shared this problematization posed by their group where they believe the low SES community seem to suffer more by environmental issues.

Another student from Group 4 echoed that this mathematical modeling task was not simply a mathematics task and connected with a structural problem of the government, stating “I could not shut off the social justice portion of my brain. As soon as we saw this problem, we were thinking about the basics, which is the daily needs of children, right.” And then, they urged the initiative from government, saying “Obviously, we are talking about Walmart. But what is the government’s involvement in this debacle?”

Table 6. Selected variables when formulating a mathematical model

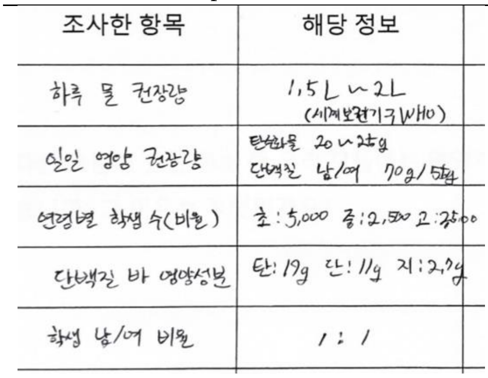
Variables		U.S. PSTs						Korean PSTs						
		G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6	G7
Average calorie/water/protein intake requirements for schoolchildren		V	V	V	V	V							V	
Average requirements for schoolchildren	by age						V							
	by biological sex									V				
	by age and biological sex								V					V
	by biological sex and weight							V			V	V		
Number of school days left		V	V	V	V	V								
Amount of protein in a protein bar		V		V				V	V	V	V	V	V	V
Number of students with a protein allergy								V						
Leap Month based on the Chinese Lunar year														V
Size of water bottle		V	V	V		V		V		V	V	V		V
Health issue	Side effects of protein intake	V	V	V	V	V		V		V				
	Water for other purposes (e.g., washing)				V					V		V		
Social justice issues (e.g., SES of the Flint people)				V	V	V	V							
Success to find outcomes		V	V	V	V	V		V			V		V	V

Mathematical Modeling Process of Korean PSTs

Selected Variables to Formulate a Mathematical Model. Korean PSTs focused on more diverse variables to formulate a model. Similar to U.S. PSTs, they considered average protein, calorie, and water intake requirements for schoolchildren of the city of Sokcho. For example, Group 6 formulated a model based on that information. However, other groups argued to consider age, biological sex, and weight of students to formulate a model.

Using data from a portal website, Group 2 found six information: (a) daily water intake requirement is 1.5 ~ 2.0 liters, (b) the ratio of students by grade levels for elementary, middle, and high school is 5 : 2.5: 2.5 (2 : 1: 1), (c) the ratio between boys and girls is 1 : 1, (d) The daily carbohydrates intake requirement is 20 ~ 25 grams, (e) The daily protein intake requirement is 70 grams for boys and 55 grams for girls, and (f) a protein bas has 11 grams of protein, 19 grams of carbohydrates, and 2.7 grams of fat (see Figure 5). Thus, Group 2 argued that the protein bar distribution for school children should be different based on their grade levels and biological sex. However, they were unsuccessful in computing their models and generating answers to the problems. As they considered carbohydrates, protein, and fat, their calculation differed according to what nutrition they focused on.

Group work sheet



조사한 항목	해당 정보
하루 물 권장량	1.5L ~ 2L (세계보건기구 WHO)
일일 영양 권장량	탄수화물 20 ~ 25g 단백질 남/여 70g/55g
연령별 학생 수(비율)	초: 5,000 중: 2,500 고: 2,500
단백질 바 영양성분	탄: 19g 단: 11g 지: 2.7g
학생 남/여 비율	1 : 1

Translated group work sheet

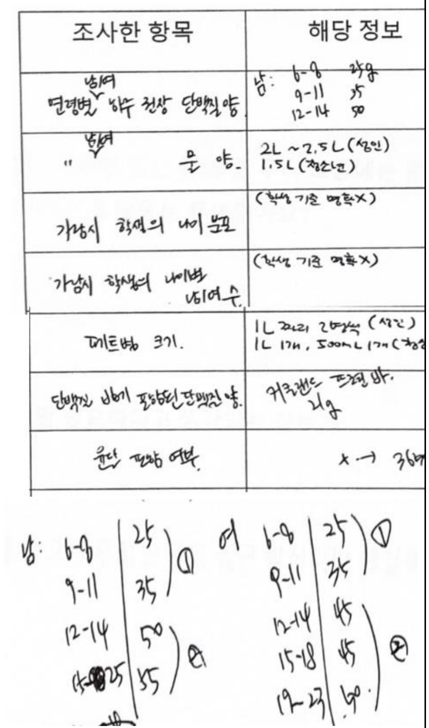
Selected variables	Relevant information
Daily water intake	1.5L ~ 2.0L recommended by World Health Organization
Daily nutrition intake requirement	Carbohydrates 20 ~ 25g Protein male/female 70g/ 55g
Ratio of student by grade levels	Elementary: 5,000 Middle: 2,500 High: 2,5000
Nutrition in a protein bar	Carbohydrates 19g Protein 11g, Fat 2.7g
Ratio between boys and girls	1:1

Figure 5. The selected variables of Korean PSTs – Group 5
Note. ‘L’ refers to liters and ‘g’ indicates grams.

Similarly, Group 7 considered the student's age and biological sex (see Figure 6). However, the included college students (from Kindergarten to 16) assume that they are also ‘students’ and need water. They also considered that as Korean males need to accomplish military service for two years, male college students are two years older than female college students. Thus, they divided age bands into four or five groups; for males from 6 to 25 and for females from 6 to 23. The size of the water bottle also differed based on student age. They assumed that college student needs 2 bottles of 1-liter bottle, and children need 1 bottle of 1-liter bottle, 2 bottles of 500-milliliter bottle. However, they did not consider the number of students by specific age bands.

The other three groups (Group 1, Group 4, and Group 5) considered the biological sex and weight of students to formulate a model. For example, Group 4 found that the average weight of 8 ~ 17-year-old students is 52 kilogram for boys and 45 kilogram for girls, and students need 1 gram of protein by 1 kilogram. Thus, they assumed that every day boys and girls need 52 grams and 45 grams of protein, respectively (see Figure 7).

Group work sheet



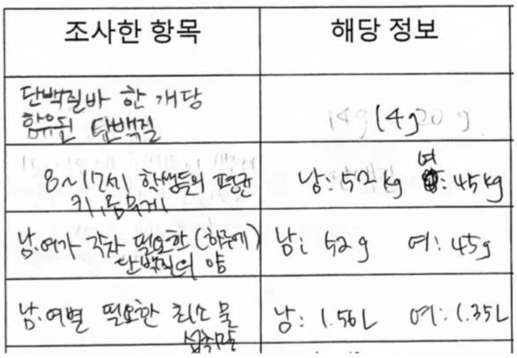
조사한 항목	해당 정보

Translated group worksheet

Selected variables	Relevant information
Daily protein intake requirements for schoolchildren by age and biological sex	Boy: 6 ~ 8, 25g; 9 ~ 11, 35g; 12 ~ 14, 50g; 15 ~ 25, 55g Girl: 6 ~ 8, 25g; 9 ~ 11, 35g; 12 ~ 14, 45g; 15 ~ 18, 45g; 19 ~ 23, 50g
Daily water intake requirements for schoolchildren by age and biological sex	College student: 2 L ~ 2.5 L Children: 1.5 L
Student age distribution	Not available
Student age distribution	Not available
Ratio between boys and girls	1:1
Size of water bottle	College student: 2 bottles of 1L bottle Children: 1 bottle of 1L bottle, 2 bottles of 500 ml bottle
The amount of protein in a protein bar	21 g (*Kirkland* protein bar)
Inclusion of Leap Month	No (365 days per a year)

Figure 6. Selected variables by Korean PSTs – Group 7
Note. ‘L’ refers to liters, ‘ml’ means milliliters, and ‘g’ indicates grams.

Group work sheet



조사한 항목	해당 정보
단백질바 한 개당 함유된 단백질	14g
8~17세 학생들의 평균 키, 몸무게	남: 52kg 여: 45kg
남·여가 각자 필요한 (하루에) 단백질의 양	남: 52g 여: 45g
남·여별 필요한 최소 물 섭취량	남: 1.56L 여: 1.35L

Translated version

Selected variables	Relevant information
Nutrition in a protein bar	14 g
Average weight of 8 ~ 17 years old students	Boy: 52 kg Girl: 45 kg
Daily protein intake requirement by sex per day	Boy: 52 g Girl: 45 g
Daily water intake requirement by sex per day	Boy: 1.56 L Girl: 1.35 L

Figure 7. The selected variables of Korean PSTs – Group 4
Note. ‘L’ refers to liters, ‘kg’ means kilograms, and ‘g’ indicates grams.

Other Considered Variables and the Stance of Social Justice. Similar to U.S. PSTs, Korean PSTs proposed to consider health issues during the third phase. Regarding water contamination, two groups suggested to provide students with more water as they need water for taking a shower and washing dishes. Group 3 stated, “Lead-poisoned water is bad for our skin, and I think we need to provide more water for the students to wash.” Group 5 also presented a similar argument stating “I don't think that bowls would be clean as the water is contaminated with lead. So we need to give them additional water to wash their bowls. And we need to give them water with 500ml bottle instead of a large size of water bottle.”

Regarding the side effects of protein intake, Group 3 was concerned about providing protein bars. Based on the internet news, they claimed that more protein intake would cause dehydration, nausea, cardiovascular disease, and blood vessel disorders. Thus, students need to be provided with other healthy food. Moreover, Group 1 discussed protein allergy that some students may have and suggested to provide them with other food rather than protein bars.

While U.S. PSTs discussed a need for distributing more protein bars as well as urged governments to take active actions given the existing barriers that hampered survival of the Flint people with low SES, none of Korean PSTs suggested to consider these measures and actions when solving the modified Flint Water task. Instead, their discussion focused on access to water and protein. Korean PSTs took a stance emphasizing fairness, as they believed that schoolchildren in Karam city would need a similar amount of water, nutrition, and protein bars as students in other cities. In sum, a disparity in approaches to a social justice issue such as the water crisis was noted between U.S. PSTs and Korean PSTs.

V. DISCUSSION

Considering the globally growing interest in mathematical modeling in K-12 mathematical curriculum, this article examined the mathematical modeling process on a social justice issue. Most previous work on PSTs’ mathematical modeling focused on a single country (Hankeln, 2020), which confines the potential benefits of international comparative studies. Our analysis contributes to existing literature by focusing on U.S. and Korean PSTs' engagement in the modified Flint water task, revealing similarities and differences in their justice-oriented mathematical modeling processes.

Prior research has demonstrated that when PSTs engage in socio-critical mathematical modeling, even without prior experience in mathematical modeling, they address the complexity of connecting mathematical modeling to social justice (e.g., Jung & Magiera, 2023). However, these studies primarily stemmed from U.S. contexts. This study incorporated Korean PSTs to investigate how they adopt the varying degrees of a social justice stance to solve a justice-oriented mathematical modeling task, broadening the understanding of international backgrounds.

To assess PSTs’ engagement in the justice-oriented mathematical modeling process, we identified and analyzed their involvement in the mathematical modeling cycle (making sense of the task, formulating a mathematical model, sharing models) and social justice cycle (real situation, broad social issue, social awareness & sense of action) by building on existing literature (Aguirre et al., 2019). What distinguishes our study from existing literature is we found that PSTs’ perception and enactment of social justice can be an undergirding drive to solve a mathematical modeling task. Future studies can delve into other sociocultural backgrounds (e.g., cultural norms, values, beliefs, interactions) that may impact the PSTs’ process of constructing mathematical models. These identifications can inform mathematics teacher educators about directing PSTs to reconsider variables and methods, encouraging them to think differently about what to consider and how to validate the accuracy of their mathematical models.

In terms of the mathematical modeling cycle, both U.S. and Korean PSTs addressed two variables – the quantity of protein bars and water bottles (or water) needed per day. However, they diverged in considering additional variables to formulate a model. U.S. PSTs took into account the number of school days and the average requirement of water, calorie, and protein for schoolchildren, whereas Korean PSTs factored in age, biological sex, and weight of students. These differences suggested that the mathematical model of U.S. PSTs was relatively simpler than Korean PSTs in terms of the number of variables considered, and Korean PSTs formulated more nuanced model considering various variables.

However, some Korean PSTs did not generate a conclusion to the task possibly due to the complexity of the models that included diverse variables (see Table 6). Korean Group 5 was one example who was unsuccessful in computing their models as they considered various nutrients in a protein bar such as carbohydrates, protein, and fat in calculating the amount of protein bars needed for schoolchildren. These findings highlight the need for further discussion in the process of teaching mathematical modeling to PSTs about the pros and cons of considering a wide range of variables. While considering various variables might help PSTs arrive at a more plausible answer closely reflects reality, they might encounter challenges in computing and drawing a conclusion.

The observed differences in engagement in the mathematical modeling cycle between U.S. and Korean PSTs also imply that PSTs' mathematical modeling process might be influenced by their lived experiences and cultural backgrounds (Suh et al., 2021). For example, in calculating the number of water bottle, U.S PSTs used ounces as a measurement unit while Korean PSTs used liters. Additionally, regarding the interpretation of the number of school days, all Korean PSTs considered it as 365 days. One group of Korean PSTs suggested to consider the leap month following the Chinese Lunar year. Conversely, some U.S. PSTs derived the number of school days as 110 or 180 days by deducting vacation days from the total annual period. In addition, one Korean group incorporated discussions about Korean males' mandatory military service to divide age bands of males (see Figure 6).

Moreover, unlike Korean counterparts, U.S. PSTs refrained from addressing biological sex differences in the modified Flint Water task. They held the perspective that dichotomizing individuals into "boys" or "girls" might not be inclusive, considering various gender identities students could identify with (Thanheiser, 2021). The currently contentious discourse on gender concerns in the U.S. might have influenced their approach to this task.

Regarding the social justice cycle, U.S. PSTs demonstrated a more active engagement in connecting the modified Flint Water task to social justice issues. When selecting variables and creating a mathematical model, they aimed to distribute more water and protein to marginalized populations. They also discussed a need for the systematic changes, urging not only companies but also advocating for government-level initiatives directed toward Flint. However, Korean PSTs did not perceive this issue as a social justice concern, although they expressed health concerns, water pollution, and empathy toward students in Sokcho.

These difference across two countries might be explained by the discourse around equity in mathematics teacher education (Gutstein & Peterson, 2013). The educational system in the U.S. emphasizes social justice concerns in K-12 mathematics education and mathematics teacher education programs (AMTE, 2017, NCTM, 2014; 2020). Conversely, in Korea, K-12 mathematics education and mathematics teacher education programs place a high value on acquiring mathematical knowledge (MOE, 2022; Hwang et al., 2003; Park, 2016). While these focuses may not be applicable to every educational situation in each country, they can influence PSTs' engagement in mathematical modeling and social justice cycles. In sum, our study shows that the cultural and educational backgrounds of PSTs influence how they interpret a real-world situation portrayed in a mathematical modeling task as well as construction of mathematical models. Future research needs to test these hypotheses by exploring the influence of PSTs' sociocultural backgrounds on the justice-oriented mathematical modeling process.

Overall, this study revealed that PSTs' stance on justice was not isolated to a social justice cycle, but was embedded in the entire modeling process across how they make sense of the task (the first phase), formulating a mathematical model (the second phase), and sharing their models (the third phase). Particularly, phase 3 was the most prominent stage where PSTs' stances on social justice issues were most visible as they bounced off ideas, clarified and justified their positions, and were influenced by other group perspectives (Jung & Magiera, 2023). Given that mathematical modeling activities typically focused on the solving the task (Maaß, 2006), not sharing the models, educators need to consider how to leverage the share-out time to deepen students' understanding of the mathematical modeling activities.

VI. CONCLUSIONS

The competency and perspective of PSTs influence their instructional practices in future mathematics classrooms and the learning experiences of their students. Considering the importance of the mathematical modeling and social justice in mathematics education, it is imperative to understand current PSTs' mathematical modeling acitivies connected with social justice issues. This study examined the U.S. and Korean PSTs' engagement in a justice-oriented mathematical modeling activity that involves mathematical modeling and social justice cycles.

The contribution of this study is that it revealed both convergence and divergence between U.S. and Korean PSTs in how they formulate a mathematical model, as well as their conceptualization of equity. It also documented cultural backgrounds and educational discourse as possible reasons that may affect the differential engagement of these two groups of PSTs in justice-oriented mathematical modeling activities. Therefore, this study enriched the field's understanding of how PSTs from different countries engage in a mathematical modeling task on a social justice issue, s topic that has been scarcely studied in existing literature.

Given the affordances of justice-oriented mathematical modeling activities documented this study, we suggest that mathematics teacher educators consider providing PSTs with opportunities to nurture their mathematical knowledge and unpack their perspectives regarding social justice through justice-oriented mathematical modeling tasks. These efforts could enhance instructional competency of PSTs and positive impacts in our society.

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