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DEVELOPMENT ARTICLE

Augmented reality for area measurement reasoning of elementary students

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Abstract
Area measurement is a foundational component across STEM fields. The area formula offers limited insight for children trying to develop their area measurement reasoning skills, particularly when dealing with composite shapes. Spatial structuring, where children explore structural units of a 2D space, is an alternative approach. However, poor manipulation of physical units can reinforce misconceptions about area measurement. To address this issue, we developed MeoGeo, an augmented reality (AR) smartphone application. MeoGeo allows children to readily create virtual structural arrays with units of varying sizes, superimposing them on their surroundings in real-time. We conducted an exploratory multiple-case study over a 7-week program to examine the effect of AR activities. Data included written responses from three elementary students on paper-and-pencil tests, along with in situ videos recordings. Our findings indicate that engagement in AR activities facilitated reasoning skills in area measurement for both basic and composite shapes. Furthermore, students maintained their reasoning skills beyond the intervention period. Our study underscores the importance of developing AR systems that align explicitly with the developmental progression of each child, and it highlights the critical role of the instructor in effective execution of AR activities.

Keywords Augmented reality · Area measurement · Mathematics reasoning · Mathematics education · Intelligent system · Elementary education

Introduction

Area measurement is an integral component of science, technology, engineering, and mathematics (STEM) education (Barett et al., 2017; Clements, 2003; Davydov, 1991; So, 2013, van den Heuvel-Panhuizen & Buys, 2008). The understanding of area measurement helps students acquire important mathematics knowledge that extends beyond geometry and measurement, including concepts such as “counting, fractions, multiplication and division, interpreting remainders, the associative and distributive properties, place value, and operations with rational numbers” (Wickstrom et al., 2017, p. 115). Thus, it is imperative to develop students’ area measurement reasoning skills.

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Young children start identifying area as an attribute of plane figures as early as ages 0–2 (Clements & Sarama, 2020), and it is expected that they will master area measurement during their elementary school years (Common Core State Standards Initiative [CCSSI], 2010). However, many students face challenges with area measurement (Battista, 2007; Baturo & Nason, 1996; Runnalls & Hong, 2020; Simon & Blume, 1994). Area measurement is the weakest skill for 4th grade students on both the 2019 and 2022 National Assessment of Educational Progress (NAEP) mathematics assessment—only 38% of students surveyed in 2022 and 44% of students surveyed in 2019 were able to correctly solve the measurement of rectangular items (NAEP, 2024). These challenges often stem from a limited skill to visualize the structural aspects of area and an overreliance on area formulas (Battista, 1982; Lehrer, 2003; Nitabach & Lehrer, 1996; Nunes et al., 1993; Outhred & Mitchelmore, 2000). Given that the mastery of area measurement and modelling lay the groundwork for success in advanced mathematical concepts, such as calculating the area under the curve for definite integrals (Jones, 2015), devising an effective instructional activity is crucial.

Researchers have suggested that instructional activities that utilize cultural tools as physical manipulatives may be effective in helping students understand the concept of area (Wickstrom, 2022; Wickstrom et al., 2017; Zacharos, 2006). Sarama and Clements (2009, p. 315) claimed that "only with high quality instruction do they (children) form generalizations about measurement across attributes. Such instruction integrates development of procedures and concepts." Effective activities for area measurement include enumeration of squares to cover designated areas with arrays of differently sized units, which facilitates mental structuring of the 2D area, called spatial structuring (Battista, 2007; Clements et al., 2017; Wickstrom et al., 2017). However, the effectiveness of physical manipulatives in supporting spatial structuring is constrained by their fixed shapes and sizes, rendering children unable to dynamically modify units (Bujak et al., 2013). Also, children's mental and manipulative skills are often limited, leading to gaps or overlaps in their attempt to impose structure onto a 2D space (Curry & Outhred, 2005).

In this study, we introduce MeoGeo. This smartphone application was designed for teaching area measurement by leveraging augmented reality (AR) technology. MeoGeo enables children to seamlessly coordinate 2D space by overlaying virtual arrays of units onto their everyday surroundings in real-time. Our approach drew from literature on developmental progressions of children in their learning of area measurement (Clements & Sarama, 2020).

MeoGeo distinguishes itself from the tools introduced in previous studies for teaching area measurement in three key aspects. Firstly, unlike physical manipulatives (e.g., tangrams) with their fixed sizes and shapes (Bujak et al., 2013), MeoGeo allows students to manipulate units, adjusting their sizes to represent both basic and composite shapes with composite units (unit of a unit such as rows and columns). This was designed to enhance their understanding of area measurement concepts.

Secondly, MeoGeo enables students to easily construct these units in the physical space of their surroundings using marker-less AR technology. This was designed to help students avoid procedural, formula-based understanding, which tends to be disconnected from their surroundings (Baturo & Nasson, 1996). Previous AR approaches based on QR markers face similar limitations, since they work with predetermined objects on a worksheet (Arican & Özçakir, 2021).

Finally, MeoGeo discretizes the physical space of the user into a regular grid, enforcing placement of virtual units. As described by Curry and Outhred (2005), students often leave gaps and overlaps when placing physical manipulatives. Enforcing placement of units on a regular grid, we eliminate these common errors.

In this study, we aim to understand the reasoning processes and skills of elementary students for basic and composite shapes in the context of instructional activities that utilize MeoGeo. The technological scope of the app is unique in that it combines marker-less technology, the easy manipulation of 2D space, and design principles centered around the learning trajectory of each child. Compared to current alternative AR tools (Arican & Özçakir, 2021; Chao & Chang, 2019; Demitriadou et al., 2020), particularly related to area measurement (Hwang et al., 2023; Korenova, 2019; Mueller & Platz, 2022), our system is the first, within the scope of our search, to explicitly incorporate children's developmental progression in area measurement (Clements & Sarama, 2020) and allow students to observe the structural elements of both basic and composite shapes through the discretization of space, changes in virtual unit square sizes, and seamless AR marker-less technology. Furthermore, our study goes further by examining the children's process of reasoning instead of simply focusing on test scores. We found that our results have important implications for the learning of area measurement as a foundational topic in STEM.

Literature review

Area measurement

Area measurement is an essential concept that students should master during the early grades of elementary school (Clements & Sarama, 2020). Area refers to the amount of a 2D region within a boundary, and area measurement involves quantifying the surface enclosed within that 2D region (Lehrer, 2003). Area measurement requires a complicated process that includes three main components: the identification of shapes and their properties, identification of measures (e.g., length), and computation of measures (Owens & Outhred, 2006).

Previous studies on area measurement have documented difficulties that arise when students solve area measurement problems solely using the computational process of area = base × height (e.g., Battista, 1982; Lehrer, 2003; Nitabach & Lehrer, 1996; Nunes et al., 1993; Outhred & Mitchelmore, 2000). Elementary students also often struggled with computing the area of composite shapes, even when they can calculate the area of basic shapes (Lehrmann, 2023). Since composite shapes consist of combinations of more than two basic shapes (e.g., Fig. 1), students cannot simply apply procedural algorithms (Patahuddin et al., 2018; Zacharos, 2006). Instead, they are required to decompose the composite shape into basic shapes or equal-sized units to find area (Spiegel & Ginat, 2017). Therefore, teachers were recommended to provide opportunities for their students to overlay small unit squares onto square grids to construct larger composite shapes (Patahuddin et al., 2018; Zacharos, 2006).

Simon and Blume (1994) suggested a two-step approach for teaching area measurement. The first step involves considering area as a quantity and then counting the number of unit squares. In the second step, students evolve their understanding into a multiplicative reasoning process, involving *n* rows and *m* columns of unit squares (Patahuddin et al., 2018). This step is known as "spatial structuring," which entails visualizing and locating

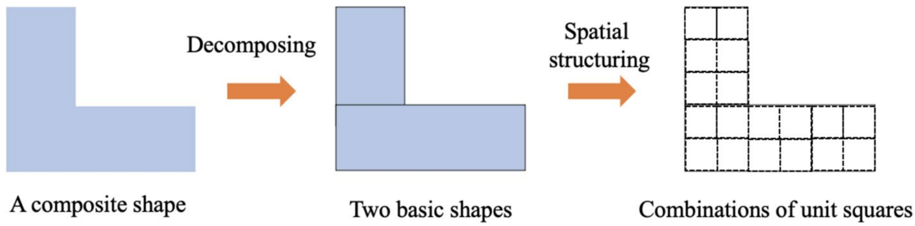


Fig. 1 Area measurement process for a composite shape

composite units (i.e., unit of unit), such as rows and columns, within a 2D space. This computational process differs from purely numerical reasoning (i.e., base × height) (Lehrer & Solvin, 2014).

Clements and Sarama (2020) proposed a learning trajectory for area measurement that explains how children conceptually understand and calculate area according additive and multiplicative reasoning. Table 1 enumerates observable behaviors and hypothesized mental actions of students in each phase of the learning trajectory.

In Phase 1 (Area sensor), children develop early conceptualizations of area measurement. In Phase 2 (Area quantity reorganizer) and Phase 3 (Physical coverer and counter), students rely on perceptual support from constructs, such as a grid, to make sense of area. They then progress to cognitive area calculation in Phase 4 (complete coverer and counter) and Phase 5 (area unit relater and repeater). In these phases, students could calculate area measurement without perceptual support, wielding their understanding of additive reasoning. Subsequently, in Phase 6 (Initial composite structurer), students can calculate area using rows and columns of different unit sizes. In Phase 7 (Area row and column structurer), they can decompose and recompose a large shape into small partial units. Finally, in Phase 8 (Array structurer), students demonstrate flexible use of multiplicative reasoning. Therefore, it is important for teachers to assess and understand the developmental phases of their students and provide appropriative support to help them progress through each phase.

The progression of student' understanding of area varies across national and state curricula. However, students are generally expected to reach phase 8 by Grades 3 or 4. In the CCSSI (2010) in the United States, for example, Grade 3 students develop an understanding of area as a measure of two-dimensional space by recognizing its conceptual meaning and measuring it through unit square counting. Additionally, they are expected to apply additive reasoning and multiplication to determine the area of rectangles. By Grade 4, students extend their knowledge by employing area formulas to solve related problems, such as determining unknown dimensions of a rectangle.

Instructional tools for area measurement

Researchers have investigated several instructional tools that support students in learning area measurement (Nunes et al., 1993; Zacharos, 2006). They reported that teaching area measurement using two dimensional units (e.g., plane figures) is more effective than employing a one-dimension tool (e.g., rulers), due to its direct relationship with the measured dimension. Among various plane figures, rectangles are particularly effective for teaching area measurement because they "tessellate the plane and fill the same space uniformly on both dimensions, thereby supporting the multiplication of lengths" to calculate

Table 1 Learning trajectory for area measurement (adopted and re-classified from Clements & Sarama, 2020)

Phase	Descriptions of observable behaviors and hypothesized mental actions of students
Phase 1. Area Senser	A child begins developing early conceptualizations of area measurement. However, he/she may not explicitly recognize area as an attribute. If asked to fill in a rectangle, he/she may draw approximations of circles
Phase 2. Area quantity recognizer	A child perceives the amount of a 2D space and can make intuitive comparisons. However, when asked to compare, he/she may compare lengths more than areas because lengths are more intuitively graspable and familiar to them, or make estimates based on a "length plus (*not times*) width." He/she may compare areas correctly if a task involves superimposing two different areas
Phase 3. Physical coverer and counter	If asked to find an area, a child attempts to cover a rectangular space with physical tiles and counts the tiles by removing them one by one. However, he/she does not structure the 2D space without considerable support. He/she may represent only certain aspects of the structure such as filling the space only next to existing guides (e.g., sides of region), leaving gaps, or only aligning in one dimension
Phase 4. Complete coverer and counter	A child draws a complete covering of a specific region without gaps or overlaps, approximating rows. However, he/she may make errors in aligning units. He/she may count units around the border, then count some in the interiors twice and skip others unsystematically
Phase 5. Area unit relater and repeater	A child counts individual units, often trying to use the structure of rows. She/he completes covering based on an intuitive notion of rows and columns, making equal-sized units that are lined up but may not see groups of units making up individual rows or columns
Phase 6. Initial composite structurer	A child identifies a square unit as both a unit and a component of a larger unit of units such as a row and column, and uses those structures in counting or drawing. However, she/he needs figural support to structure the space themselves. This may include physical motions of some of the tiles. He/she usually does not coordinate the width and height
Phase 7. Area row and column structurer	A child decomposes and recomposes partial units to make whole units. He/she may draw rows as rows making parallel horizontal lines and so forth. He/she reasons about conserving area and additive composition of areas (e.g., how regions that look different can have the same area measure)
Phase 8. Array structurer	A child multiplicatively iterates rows or columns to determine the area. He/she conceptually understand the rectangular area formula. He/she understands and justify that differently-shaped regions can have the same areas

area (Smith et al., 2016, p. 240). Empirical studies have reported the effectiveness of using rectangles for teaching area measurement (e.g., Battista, 1982; Bunt et al., 1986; Nitabach & Lehrer, 1996). For example, Zacharos (2016) reported that the overlapping strategy, which employs a rectangle as a basic unit square, could improve reasoning skills for area measurement.

To provide these learning experiences, researchers have suggested the use of physical manipulatives such as color tiles, rulers, tangram, and pattern blocks. While these materials can help students perform shape identification and area computation, they may offer relatively limited learning experiences due to their fixed physical characteristics. Additionally, when students interact with physical manipulatives, another major downside is that the manipulation of multiple pieces (e.g., pattern blocks) generates an excessive cognitive load, which leads them to lose focus on the intended mathematical concept of the lesson (Suh & Moyer-Packenham, 2008). Recently, researchers have investigated technology-based manipulation tools utilizing AR and virtual reality (VR) affordances to allow students to interact with diverse symbols while reducing cognitive loads (Bujak et al., 2013; Flavin & Flavin, 2024).

The characteristics of AR

Both AR and VR connect between graphical representations of geometric figures and their symbolic and numeric representations (Arican & Özçakir, 2021). However, VR is limited in interactivity compared to AR, as the interface is confined to the virtual screen (Bouck et al., 2015; Moyer et al., 2002). As illustrated in Fig. 2, AR distinguishes itself from VR by enhancing real-world views with virtual environments, thereby overcoming these limitations. In AR-based learning environments, students can have immersive experiences, collaboratively working with virtual objects (e.g., mathematical symbols and signs) projected onto devices within their actual surroundings (Bujack et al., 2013; Flavin & Flavin, 2024). Thus, AR serves as the intermediary between the virtual environment and real world, providing a medium for learning and teaching (Milgram et al., 1995).

Research has shown that AR applications positively impact K–12 student learning in geometry compared to traditional methods (Cai et al., 2020; Chang et al., 2022; Kaufmann, 2002). This can be attributed to AR affordances that support the understanding of abstract mathematical knowledge and skills for students (Li et al., 2021). Bujak et al. (2013) identified three key benefits of using AR for mathematics education. Firstly, AR offers embodied representations, thereby enhancing spatial understanding and the retention of mathematics content. Secondly, AR synchronizes information in the appropriate time and space, presenting concrete mathematical concepts. Lastly, AR enables contextually relevant learning environments that integrate in-person interaction with virtual content. Flores-Bascuñana et al. (2019) also highlighted the potential benefits of AR for interactive learning, enhancing cognitive, spatial, and motor skills.

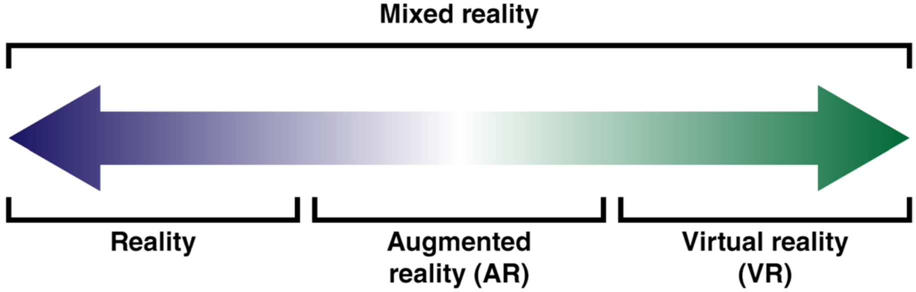


Fig. 2 Reality-virtuality continuum

AR tools are a powerful didactic resource that enhances learning experiences (Fernández-Enríquez & Delgado-Martín, 2020), but the roles of instructors during learning activities are also critical. Their communicative strategies help students understand mathematics concepts and skills beyond simple manipulation of technological devices. For example, Abrahamson et al. (2014) described several responsive teaching strategies when using a touch screen device: (a) eliciting and probing ideas, (b) summarizing and offering interpretation of ideas, and (c) elaborating on ideas to engage learners in multimodally expressed, embodied contributions. These strategies supported students in their mathematical investigation and reflection, resulting in the development ofWe have strengthened the literature by incorporating previous studies that highlight the importance of combining explicit instruction with AR tools. mathematical knowledge. Mueller et al. (2022) found that explicit instruction assisted by AR led to higher student achievement in mathematical reasonability than experiential learning assisted by AR, which involves less teacher intervention. Similarly, Morris et al.'s (2022) study on students with disabilities found that an intervention combining explicit instruction, augmented reality, and video modeling improved mathematical skills. Thus, instructors need to foster and guide student mathematics learning when they interact with AR tools.

Conceptual framework: social scaffolding and technological scaffolding

The current study adopted a conceptual framework proposed by Pea (2018), which connects social and technological scaffoldings. Scaffolding refers to "the process where a child or novice could be assisted to achieve a [certain] task that they may not be able to accomplish on their own" (Lajoie, 2005, p. 542). Social scaffolding relates to learning experiences guided by adults, teachers, or more knowledgeable peers. Meanwhile, technological scaffolding concerns learning experiences originating from the use of technological tools.

Our study posits that an instructor-child interaction (social scaffolding) and AR application (technological scaffolding) could work synergistically to enhance the cognitive development of children (Pea, 2018). AR applications could allow children to examine various mathematical shapes (both regular and irregular shapes) to calculate area measurements. Through this process, the instructor guides and supports their investigations. This scaffolding can then be removed when a child accomplishes the desired performance and goals (Pea, 2018). Once learning has been internalized, providing a scaffold is unnecessary and ineffective for maintaining knowledge and skills.

Therefore, we designed this study to assess the impact of using the AR app (technological scaffolding) with the support of an instructor (social scaffolding) on the reasoning skills in area measurement (desired performance) of elementary students during the intervention stage. Additionally, we investigated whether these students could maintain their reasoning skills after withdrawing AR activities, referred to as 'fading,' during the maintenance stage. Our approach (Fig. 3) was designed to reveal the development of elementary students and their area measurement reasoning skills over time during intervention with the AR app.

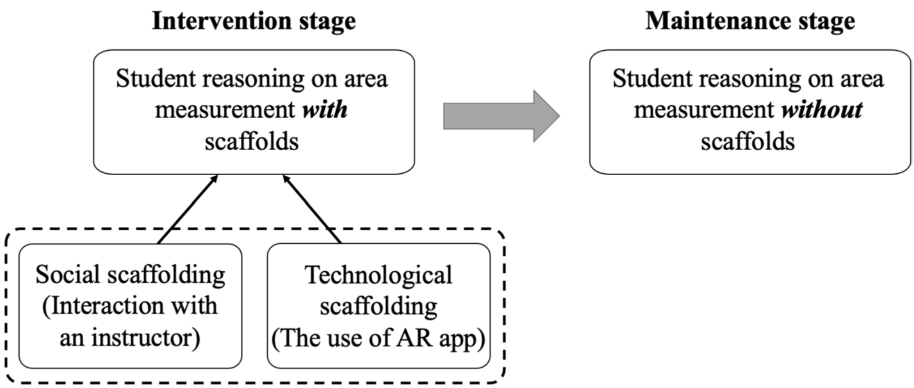


Fig. 3 Conceptual framework of this study

The current study

Previous studies have examined the influence of AR on student learning in geometry. However, most studies have focused on the identification of geometric shapes, such as the characteristics of regular shapes and volume (Arvanitaki & Zaranis, 2020; Chao & Chang, 2019; Demitriadou et al., 2020; ibili et al., 2020) and solid nets (Arvanitaki & Zaranis, 2020). There is a paucity of research examining the development of elementary students' reasoning skills for area measurement for both basic and composite shapes through AR activities. Guided by our conceptual framework (Pea, 2018), we employed AR activities as a technological scaffolder and instructors as a social scaffolder, examining how AR activities in mathematics lessons facilitate reasoning skills in area measurement. In particular, we utilized the learning trajectories for area measurement (Clements & Sarama, 2020) to examine the development of these reasoning skills. The following research questions (RQ) were investigated:

RQ 1. How do the reasoning skills of elementary students for area measurement problems change across the baseline stage (one pre-test), intervention stage (two intermediate-tests), and maintenance stage (two post-tests) following the implementation of AR activities?

RQ 2. How can engagement in AR activities with an instructor facilitate the development of the reasoning skills of elementary students on area measurement problems?

Methods

Research design

We employed an exploratory multiple-case study approach (Yin, 2009) to investigate the development of elementary students' reasoning skills in area measurement through

Table 2 Information of research participants

Variable	Robenson	Myrlène	Daisy
Age (years)	11	10	8
Grade	Rising 6th grade	Rising 4rd grade	Rising 3rd grade
Gender	Male	Female	Female
Race/Ethnicity	Black/Haitian	Black/Haitian	Black/Haitian
Home language	English and Haitian Creole	Haitian Creole	English, Spanish, and Haitian Creole
Familiarity with the use of iPad	Somewhat familiar	Somewhat familiar	Very familiar
The level of mathematical proficiency	Low	Low	Average

All names are pseudonyms. Research participants identified their familiarity with the use of iPad. The level of mathematical proficiency was identified based on the participant's school evaluation of the participant's mathematics performance

AR activities. Each participant represented a distinct case, making the multiple-case study method was the appropriate choice for this research.

Recruitment procedure, participants, and instructors

The first author recruited research participants through her partnership with a STEM education non-profit organization and conducted the current study during a 7-week summer mathematics program. The director of the non-profit organization contacted ten different schools in the two cities in the northeastern area of the U.S. From the ten students who originally expressed interest, three elementary students were selected for this study based on the following inclusion criteria: (a) Pre-test results of area measurement indicate that their area measurement reasoning skills are weaker compared to peers of a similar age (Clements & Sarama, 2020); and (b) they possess adequate motor skills to operate an iPad. Detailed participants information are provided in Table 2.

Research process

The researchers' roles were both distinct and collaborative. The AR app (named MeoGeo) was developed by the first and fourth authors of this study. Additionally, during the intervention stage, three instructors, including the first author and two undergraduate research assistants, taught area measurement using the AR app. The second and third authors were not present at the research site during data collection and focused on data analysis. They analyzed the collected data without any acquaintance with the research participants.

Figure 4 shows the research process for this study, comprising three stages: (a) baseline, (b) intervention, and (c) maintenance. The study was carried out over six weeks at a classroom hosted by the non-profit organization, followed by one week in a college conference room. Each session occurred once a week.

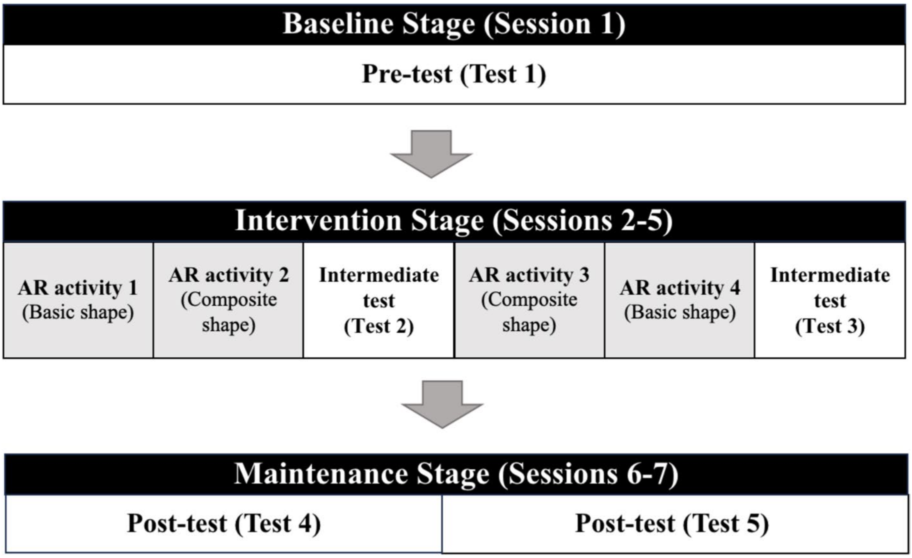


Fig. 4 Research process

In the baseline stage (session 1), participants completed a pre-test (Test 1) to assess area measurement skills before interventions. During the intervention stage, each student learned how to use an AR app, such as tapping to display a virtual unit and understanding its meaning, as well as how to overlay those units on real objects, from the three instructors including the first author and her two research assistants. The two intermediate tests were administrated respectively: the first intermediate test (Test 2) was employed after the first two AR activities in sessions 2 and 3, and the second intermediate test (Test 3) was deployed after the subsequent two AR activities in sessions 4 and 5 to measure their development of reasoning skills in area measurement. In the maintenance stage, two post-tests (Tests 4–5) were administered to examine whether the acquired reasoning skill would be retained when social and technological scaffoldings faded off (Pea, 2018). All tests were paper-and-pencil-based, implemented without the assistances of instructors and the AR app.

Development and affordances of MeoGeo

The first and fourth author developed MeoGeo, an iOS AR app, using the Swift programming language. Through ARKit and RealityKit, the app makes use of advanced sensors found in modern smartphones, including LiDAR and inertial measurement units. MeoGeo employs real-time marker-less AR technology. In this study, all students interacted with a 6th-generation iPad Pro.

The app provides various affordances, allowing students to manipulate virtual units, superimposed onto the camera feed (see Table 3). When a user (student) taps the iPad screen with their finger, a virtual unit appears in the app. The user can proportionally change the size of the unit by dragging a side length of the virtual unit. For example, a one-by-one unit can be enlarged to a one-by-two unit, two-by-three unit, etc. All virtual objects remain anchored to the physical space, as projected from the camera feed. If, for

Table 3 Four AR activities during the intervention stage

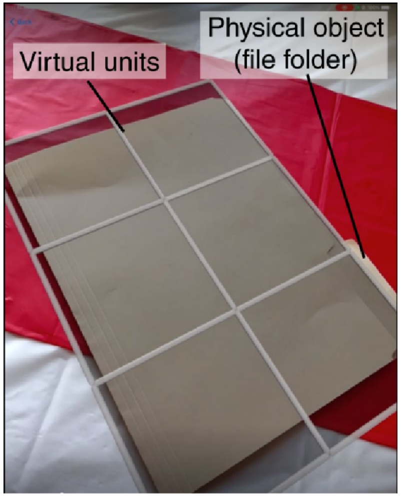
Activity number	Main task	Screenshots of iPad
Activity 1 Measuring a rectangular with two different unit sizes	Measuring a rectangle of 600 cm^2 (20 cm × 30 cm) using 10 cm × 10 cm and 10 cm × 20 cm unit sizes	

Table 3 (continued)

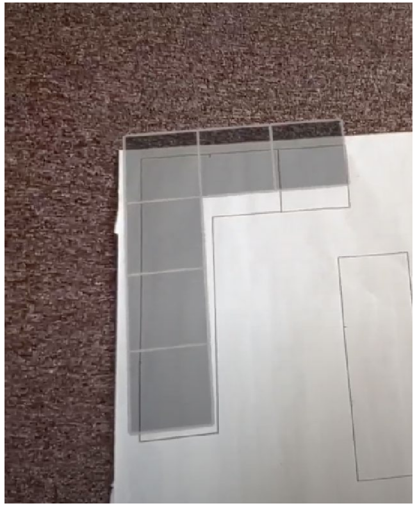
Activity number	Main task	Screenshots of iPad
Activity 2 Measuring a L-shape with two different unit sizes	Measuring a L-shape with areas of 600 cm^2 using 10 cm × 10 cm and 10 cm × 20 cm unit sizes	

Table 3 (continued)

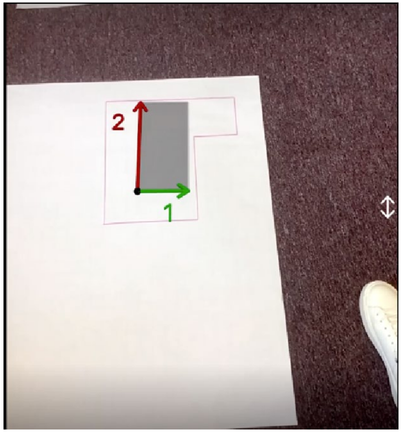
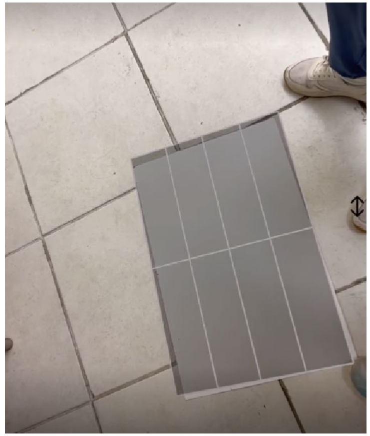
Activity number	Main task	Screenshots of iPad
Activity 3 Measuring a L-shape with three different unit sizes	Measuring a L-shape with areas of 700 cm^2 using 10 cm × 10 cm, 10 cm × 20 cm, and 10 cm × 30 cm unit sizes	

Table 3 (continued)

Activity number	Main task	Screenshots of iPad
Activity 4 Measuring a rectangular with four different unit sizes	Measuring a rectangle of 2400 cm^2 (40 cm × 60 cm) using 10 cm × 10 cm, 10 cm × 20 cm, 10 cm × 40 cm, and 10 cm × 60 cm unit sizes	

Note. Activities 1 to 4 were implemented from week 2 to week 5. Gray-colored rectangles are virtual units provided by MeoGeo example, the virtual unit is placed on top of a physical object, it will stay attached to that location even when the camera is moved back and forth. In order to provide intuitive perspective of the virtual objects, the app applies computational occlusion to hide parts of the virtual object that appear behind physical objects.

AR activities during the intervention stage

Our goal was to allow users to visualize measurable area through a virtual rectangular array of units, a technical scaffolder (Pea, 2018), to facilitate an understanding of area measurement in relation to spatial structuring. This approach aligns with the highest phase (*phase 8. Array structurer*) of the learning trajectory for area measurement (Clements & Sarama, 2020). In each intervention session, the instructor verbally explained the AR activity that a student would be working on. Following the explanation, the instructor handed out an iPad to the student and stood beside them to observe their behavior. The instructor, a social scaffolder (Pea, 2018), guided the students to think about what mathematical reasoning strategies they could use to solve the task. Given the intuitive interface design of the AR software, the students did not require detailed technical assistance during the execution of the tasks.

The intervention stage included four AR activities (one per session), and each intervention session lasted about 60 min (see Table 3). Understanding the same area can be tiled with different shapes and measured using various unit squares is essential for developing area measurement competency (CCSSI, 2010; Clements & Sarama, 2020). Therefore, we intentionally instructed students to use different unit sizes when measuring the area of shapes.

The first activity aimed for students to measure the area of a 600 cm^2 rectangle (20 cm × 30 cm) using two different sizes of units (10 cm × 10 cm and 10 cm × 20 cm). Students were prompted to discern the quantity of necessary virtual squares to cover a larger scanned rectangle. This activity sought to help students perceive both unit squares (10 cm × 10 cm) and composite units (10 cm × 20 cm) as measures of area, developing their understanding of area calculation.

The second activity focused on measuring the area of a composite shape. An L-shape was chosen for this task as past researchers and authors of large-scale assessments had utilized such configurations to assess students' skills in measuring the area of composite shapes (Baturo & Nason, 1996; Lehmann, 2023). This task prompted students to use two different-sized units (10 cm × 10 cm and 10 cm × 20 cm) to cover the L-shape. Students were expected to visually and interactively perceive that it was necessary to decompose the shape into its two basic shapes.

The third activity involved measuring the area of another L-shape with an area size of 700 cm^2. Students were asked to use an additional unit (10 cm × 30 cm) along with the previously used units (10 cm × 10 cm and 10 cm × 20 cm). Using the larger unit (e.g., 10 cm × 30 cm), students could cover more area with fewer units. Thus, they might have noticed that using a larger unit square was anefficient and effective strategy for minimizing the number of units to cover the composite shape. In the fourth activity, students were asked to measure a rectangle of 2400 cm^2 (40 cm × 60 cm) using four different-sized units (10 cm × 10 cm, 10 cm × 20 cm, 10 cm × 40 cm, and 10 cm × 60 cm).

Data collection

We collected student responses and classroom video recordings data. The student response data included their written responses to the five tests and verbal explanations of their mathematical reasoning. Each test lasted ten to fifteen minutes, followed by individual audio-recorded explanations in a one-on-one setting with one of the instructors (about 10 min).

The classroom video recording data were collected during the intervention stage to capture how students engaged in AR activities to learn the concepts of area measurements and solve relevant problems. Each video lasted approximately 60 min. We utilized three iPads in the classroom to record reasoning skills. One iPad recorded how students interacted with the AR app using screen-recording functionality, which also captured their verbal expressions. The other two iPads placed at the corners of the classroom recorded how students moved and interacted with instructors.

Area measurement test

This study utilized five tests to evaluate the development of reasoning skills in area measurement of each student. Table 4 outlines the structure of these tests. The pre-test (Baseline stage, Test 1) and intermediate-test (Intervention stage, Tests 2 and 3) consisted of three problems, respectively. The first and second problems were related to a basic shape, whereas the third problem was connected to a composite shape.

The two basic-shape problems asked students to calculate the area of a rectangle. The first problem required students to use the unit iteration strategy to identify structural arrays, such as creating rows and columns with small tile sizes to determine the area. The second problem involved finding the area using the base and height lengths. The composite shape problem (the third problem) assessed students' area measurement skills using an L-shape.

The test structure remained consistent across the three tests (Tests 1, 2, and 3). However, each post-test during the maintenance stage (Tests 4 and 5) featured a single problem. Test 4 tasked students to calculate the area of a L-shape. Test 5 required students to construct two different rectangles to represent the area defined in the question (e.g., draw two different rectangles, an area of 16 cm^2), necessitating the determination of unknown side lengths. Due to time constraints during the maintenance phase, each maintenance test covered either basic shapes or composite shapes. We provided a detailed description of sample problems of area measurement in the Appendix.

Table 4 The structure of tests

Stage	Test	The number of problems	
		Basic shape (rectangular problem)	Composite shape (L-shape problem)
Baseline	Pre-test (Test 1)	2	1
Intervention	Intermediate-test (Test 2)	2	1
	Intermediate-test (Test 3)	2	1
Maintenance	Post-test (Test 4)	0	1
	Post-test (Test 5)	1	0

Data analysis

Using a content analysis method (Hsieh & Shannon, 2005), we analyzed the collected data in five steps. First, using students' written response data, the first and second authors assessed the accuracy of students' answers to each problem. The correct and incorrect answers were coded as 1 and 0, respectively. Then, we recorded the overall score for each test. Second, using students' written responses and verbal explanation data, the first and second authors independently coded their reasoning skills to identify the phase of reasoning skills in area measurement based on Clements and Sarama's (2020) framework of learning trajectory. For example, when students measured the area of a shape with additive reasoning, their reasoning skills were classified as level 5 (see Table 1). Third, we compared the coding results to check the reliability. Interrater agreement (IRA) was calculated as agreements divided by agreements plus disagreements, multiplied by 100 (Kratochwill et al., 2010). The average IRA across five tests for accuracy was 98.86% (92–100%), and for the phase of the learning trajectory for area measurement was 92.86% (75–100%).

Fourth, the first and third authors analyzed verbal expressions and actions captured in the video recordings to examine their interactions with MeoGeo (technological scaffolding) and interaction with an instructor (social scaffolding). The authors watched the audio and video recordings repeatedly, transcribed them, and examined noticeable verbal expressions (e.g., "A student elicits help from an instructor," "An instructor asks a student to clarify her/his reasoning skills," and "A student manipulates the AR app to change the unit size"). The authors also examined the cognitive and mental processes of students based on their behaviors, such as "A student uses additive reasoning to count individual unit squares," "A student applies multiplication reasoning by multiplying rows and columns," and "A student decomposes composite shapes into basic shapes." Then, focusing on the roles of scaffolders (who enabled scaffolding), the first and third authors examined how these interactions influence the development of area measurement reasoning skills.

Results

The development of reasoning skills in area measurement

For RQ1, we examined the impact of AR activities on student reasoning skills based on their written and verbal responses to the test. Table 5 shows the accuracy of student responses (correct or incorrect) in solving area measurement problems before the intervention (baseline stage, Test 1), during the intervention stage (Tests 2 and 3), and after the intervention (maintenance stage, Tests 4 and 5). Table 6 represents the developmental phases of area measurement reasoning skills (Clements & Sarama, 2020). The findings indicated that students' reasoning skills improved when the intervention session started. However, it took more time to solve composite shape problems accurately than basic shape problems. Below, we provided a more detailed explanation of the reasoning skill development across three students.

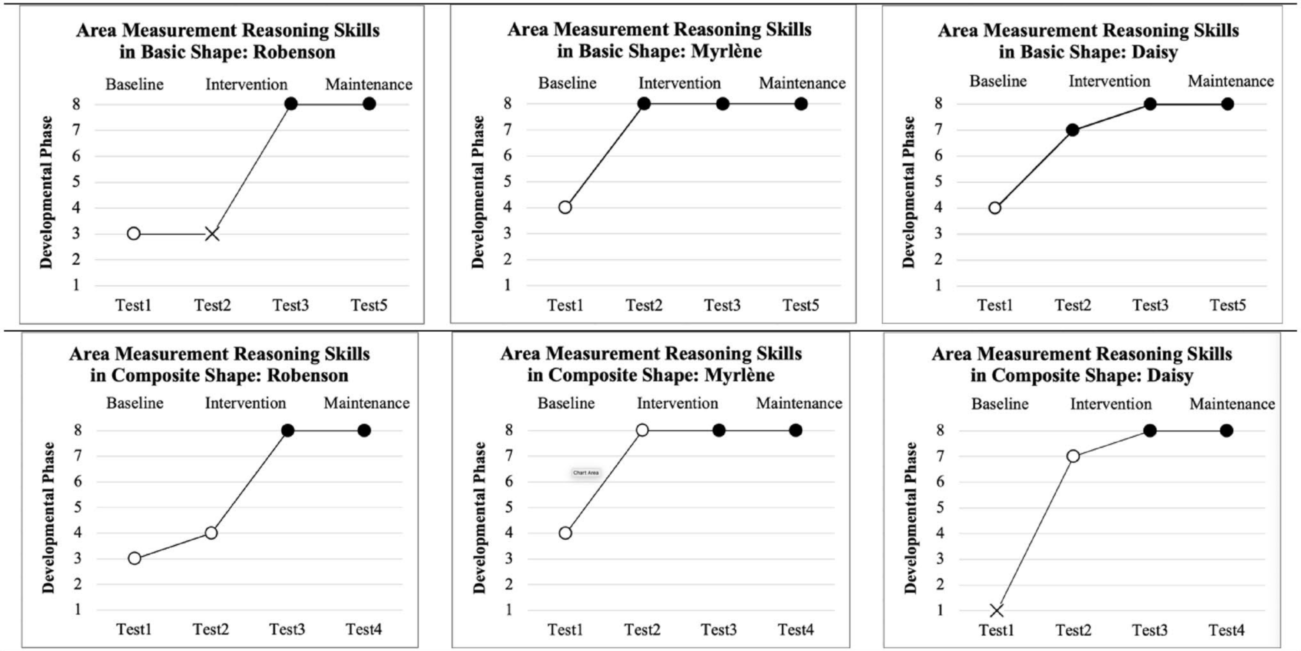
In the baseline stage (Test 1), none of the students correctly solved both basic and composite shape problems. This implies that students entered the intervention stage with a limited understanding of area measurement. Their responses to Test 1 also revealed low developmental phases in area measurement reasoning skills (see Table 7). For the basic

Table 5 Accuracy of solving area measurement problems

Stage	Test number (number of problems in the test: basic shape, composite shape)	Robenson		Myrlène		Daisy	
		Basic shape	Composite shape	Basic shape	Composite shape	Basic shape	Composite shape
Baseline	Test 1 ($n=2, 1$)	0	0	0	0	0	0
Intervention	Test 2 ($n=2, 1$)	0	0	2	0	2	0
	Test 3 ($n=2, 1$)	2	1	2	1	2	1
Maintenance	Test 4 ($n=0, 1$)	–	1	–	1	–	1
	Test 5 ($n=1, 0$)	1	–	1	–	1	–

If a student's response was correct, it was coded as '1'; otherwise, it was coded as '0'

Table 6 The development of reasoning skills based on the learning trajectory for area measurement in basic shape and composite shape (Clements & Sarama, 2020)



'○' refers to 'inaccurately solved a problem,' '●' indicates 'accurately solved a problem,' and '×' indicates 'could not explain reasoning to solve a problem at all.'

Table 7 Student response to the basic shape in Test 1: 'how many tiles would cover the entire area? show or explain your reasoning.'

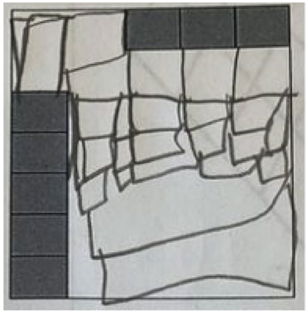
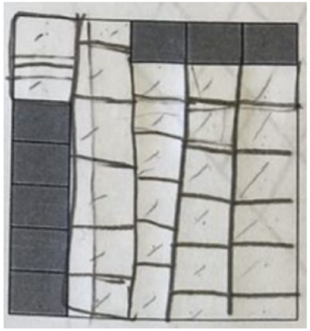
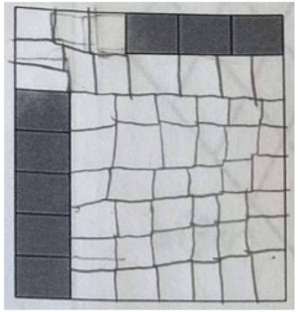
Name	Robenson	Myrlène	Daisy
Written response			
Developmental phase	3	4	4
Correctness	Incorrect	Incorrect	Incorrect

Table 8 Student's response to the composite shape in Test 2: 'find the area of the shape.'

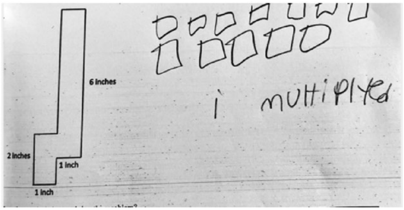
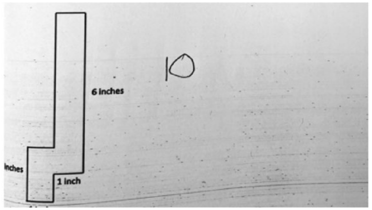
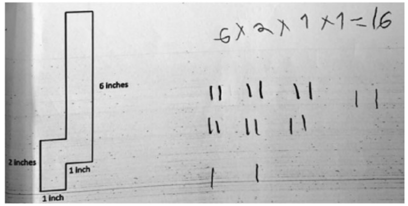
Name	Robenson	Myrlène	Daisy
Written response			
Developmental phase	4	8	7
Correctness	Incorrect	Incorrect	Incorrect

Table 9 Robenson's responses to the basic and composite shape problems in Test 3: 'find the area of the shape.'

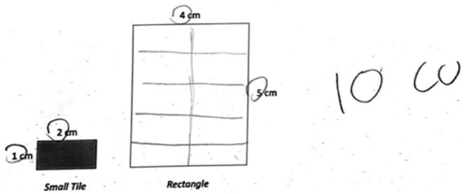
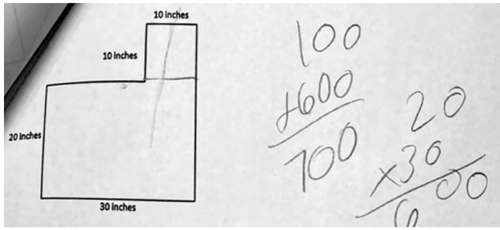
Type	Basic shape problem	Composite shape problem
Written response		
Developmental phase	8	8
Correctness	Correct	Correct

Table 10 Myrlène's response to composite shape problem in test 4: 'find the area of the shape.'

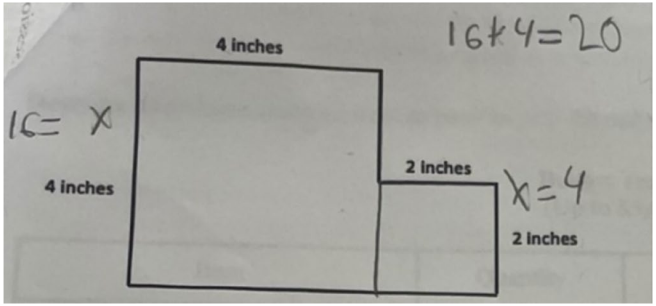
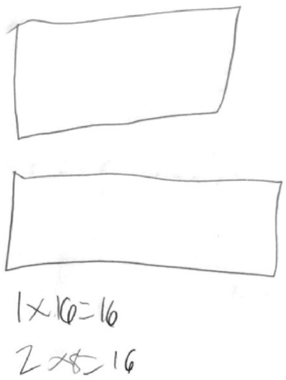
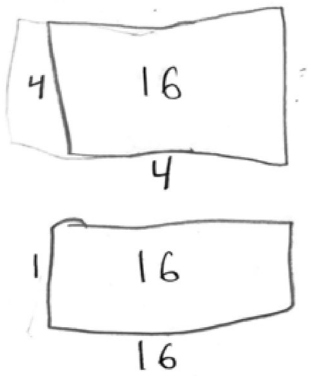
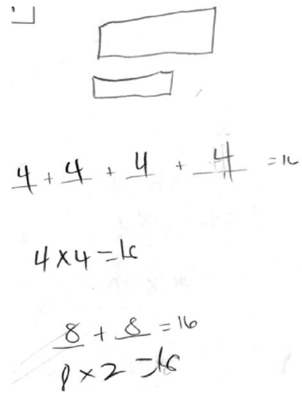
Name	Myrlène
Written response	
Developmental phase	8
Correctness	Correct

Table 11 Student responses to test 5: 'draw two different rectangles, each with an area of 16 squared centimeters. show or express your reasoning.'

Name	Robenson	Myrlène	Daisy
Written response			
Developmental phase	8	8	8
Correctness	Correct	Correct	Correct

shape problem, Robenson covered only a specific part of the shape (developmental phase 3). Myrlène and Daisy covered the entire region, but there were errors in aligning rows and columns (developmental phase 4). For the composite shape problem in Test 1, all of them showed low developmental phases (3 for Robenson or 4 for Myrlène). Daisy tried to solve a problem but could not explain her reasoning. Therefore, we coded her phase as × on the corresponding graph in Table 6.

In Test 2, following the first two AR activities, Myrlène and Daisy showed the improvement in area measurement reasoning skills (8 for Myrlène, 7 for Daisy) and correctly solve the basic shape problems. However, their answers to the composite shape problem were incorrect (see Table 8). For the composite shape problem, Robenson attempted to solve it by iteratively counting the number of unit squares that could fit, but he was unable to find the correct solution. Myrlène calculated the perimeter of an L-shape (rather than an area) by summing all the side lengths ($6+2+1+1$). Daisy multiplied all the side lengths ($6\times2\times9\times7$). These findings suggest that, although students could solve basic shape problems using systemic structural reasoning, they exhibited misconceptions when faced with composite shape problems (Lehrmann, 2023).

In Test 3, which took place after AR Activities 3 and 4, all students were able to correctly solve basic and composite shape problems. For example, regarding basic shape problems, Robenson recognized the length of each side of the small tile (Table 9, left diagram) and compared them with those of a larger rectangle. He used small tiles to multiplicatively iterate rows and columns to find the area of the larger rectangle, which shows his reasoning skills matched to developmental phase 8 (array structurer). Similarly, Myrlène and Daisy used a small tile as a measurement tool to find the area of the larger rectangle.

Regarding a composite shape problem in Test 3, all students employed the decomposition and recomposition strategies (Spiegel & Ginat, 2017). For example, Robenson (see Table 9, right diagram) split the composite shape into one 10 cm × 10 cm square and one 20 cm × 30 cm rectangle, calculating partial sums to find the area of the entire composite shape. The other two students used the same strategy. Thus, we concluded that students reached developmental phase 8 (array structurer) for both types of shapes.

During the maintenance tests (Tests 4 and 5), all participants showed reasoning skills at developmental phase 8 (array structurer). Their responses to Tests 4 and 5 indicated that they correctly solved the basic and composite shape problems using decomposition and recomposition strategies (see Table 10). In Test 5, all students obtained correct answers (see Table 11) for a problem determining rectangle side lengths from its area ($16\ \text{cm}^2$). All of them showed they conceptually understood the rectangular area formula and justified

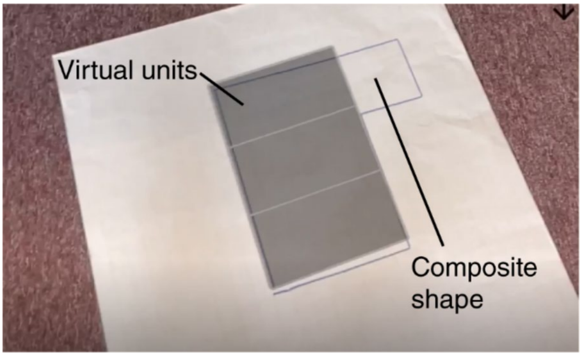


Fig. 5 Daisy's iPad screen recording of overlapping 10 cm × 20 cm gray-colored virtual units afforded by MeoGeo onto a scanned composite shape drawn on a paper

that two differently shaped regions can have the same area of 16 cm^2. These findings indicate that student reasoning skills in area measurement was maintained even after the scaffoldings were removed.

Student engagement in AR activities: the development of reasoning skills through technological and social scaffolding

This section summarizes key findings obtained from video recordings collected during the intervention stage, addressing RQ2. During the first and second AR activities, students started to perceive the area as consisting of equally sized units, matching how the AR app shows virtual unit squares. For example, Robenson, who had not initially used an equal unit square size (see Table 7), began to draw equal unit squares to determine the area (see Table 8). Although he provided an incorrect answer in Test 2, he used equal unit squares to measure the area. The other two students got the correct answer for the basic shape in Test 2 by covering the region with unit squares, which shows that AR Activities 1 and 2 benefited them in solving basic shape problems.

The third AR activity focused on composite shapes. Since the none of the students were able to correctly solve the composite shape problem in Test 2, the instructors aimed to help them understand decomposition and recomposition heuristics through relevant AR activities. Using the functionality of the AR app, students manipulated various unit sizes and the instructor guided them in covering a portion of the composite shape with 10 cm × 20 cm unit sizes, omitting a 10 cm × 10 cm section (see Activity 3 in Table 3). Interaction with the visual affordances of the AR app helped students recognize the need to break down the shape into its constituent components (decomposition). They then calculated the area of each component and added up partial sums (recomposition).

For example, in Test 3, Daisy attempted to cover a composite shape with three 10 cm × 20 cm unit (see Fig. 5). She walked around the paper, viewed these units from different angles, and tried to overlap the remaining 10 cm × 10 cm part with the 10 cm × 20 cm unit. When the instructor asked if she could cover it, she replied, “no,” and then calculated the area by summing up the areas of three 10 cm × 20 cm units and the remaining 10 cm × 10 cm part. She also noted that the 10 cm × 10 cm part was half the size of the 10 cm × 20 cm unit. Consequently, unlike her responses to Tests 1 and 2, she broke down the composite shape into different constituent shapes to calculate the area. The other two students showed similar interactions with the app and the instructor.

The fourth AR activity asked students to cover a large rectangle (40 cm × 60 cm) using four different-sized virtual units (10 cm × 10 cm, 10 cm × 20 cm, 10 cm × 40 cm, 10 cm × 60 cm). This activity encouraged students to explore efficient computation strategies: measuring the area by using one 40 cm × 60 cm unit rather than employing an enumeration strategy where students use 24 groups of 10 cm × 10 cm units. As a social scaffolder, the instructor, posed a series of questions to support multiplicative reasoning of the students. For instance, during a dialogue between Robenson and the instructor:

(Robenson tapping the screen and created a virtual array of 40 cm × 60 cm).

Instructor: How many unit squares of 10 cm × 10 cm are there?

Robenson: (Counting each square) Maybe, fifteen?

Instructor: If we don’t count one by one, what alternative method can we employ?

Robenson: (Directing his gaze at the screen) Uhm. We can multiply. There are four rows (pointing to the virtual rows with his finger). And six columns (pointing to the virtual columns then taking a brief pause) So … (pause) it [area] is 2400.

The verbal exchanges above demonstrate how Robenson's reasoning skills evolved through the interaction with the AR app and his instructor. He initially counted individual virtual units to systematically iterate rows and columns. Then, the instructor encouraged him to explore alternative methods to find the area. The virtual rows and columns on the screen aided him in recognizing the spatial structure of the array. Consequently, he coordinated the 4 rows and 6 columns as the base and height of the area and multiplied the two numbers to calculate the area of the scanned object.

After this dialogue, the instructor assessed Robenson's multiplicative reasoning skills. She guided him to use another unit size (10 cm × 60 cm) available in the app, helping him relate the 10 cm × 60 cm unit to a row of that region. Robenson linked each row by groups (4 groups of 10 cm × 60 cm) to the structure of a rectangular array of the scanned paper (40 cm × 60 cm). This example highlights how the reasoning skills of this student evolved into a more strategic process of calculating area through social interactions with the instructor and AR app.

The student responses to Test 3, following AR Activities 3 and 4, demonstrated that all students were capable of coordinating the elements of a 2D region and applying multiplicative reasoning. Furthermore, they all correctly solved the composite shape problems. These results suggest that the series of AR activities that involved the interactions with the AR app and the instructor facilitated the recognition of the spatial structure of arrays, the coordination of the number of rows and columns as width and height, and the utilization of a multiplication strategy to determine the area for both basic and composite shapes.

Discussions

The current study develops a novel AR platform (MeoGeo) and uses it for mathematics instruction to enhance reasoning skills in area measurement of elementary students. The app and instructors helped elementary students to understand the concept of area using various sized virtual units, showing them effective strategies for area measurement (Nunes et al., 1993; Zacharos, 2006). An important affordance of MeoGeo is the easy manipulation and alignment of differently sized units, which allowed students to flexibly explore structural elements, such as rows and columns, of area.

This study expands on the findings from prior literature that found teaching area measurement as 'spatial structuring' using physical manipulatives could be effective (Patahuddin et al., 2018; Simon & Blume, 1994). Our study also provides new insights into the values of using AR technology for teaching area measurement. Data from our video recordings shows the students manipulating various virtual units, which helped them to conceptually understand area measurement. AR technology presents abstract mathematical concepts more concretely, allowing students to manipulate virtual content that has contextual information from their physical surroundings (Bujak, 2013). Therefore, MeoGeo facilitates visualization of a 2D space as a structural array (Wickstrom, 2022; Wickstrom et al., 2017; Zacharos, 2006).

This study also highlights the advantage of considering children's learning trajectory of area measurement (Clements & Sarama, 2020) in evaluating the effect of virtual interactions on the mathematics reasoning skills of children. This approach enabled us to provide insight into the "process" of developmental progressions, unlike previous studies that only focused on technological feature of AR app (e.g., Chang et al., 2022) or whether students got the correct or incorrect answer (Arican & Özçakir). Considering learning trajectories, we were able to examine not only students' accuracy on area measurement problems but also the developmental progressions of their reasoning skills.

We also found that students' developmental phases persisted even after the intervention sessions were withdrawn. When new technology is introduced for mathematics learning, the focus typically revolves around one-time measurements for accuracy in mathematics achievement. Moreover, the timing of these measurements tends to occur right after the intervention. However, this study goes beyond merely assessing student test results in terms of accuracy using a one-time test. It demonstrates their developmental progression in area measurement reasoning skills over seven weeks and after the intervention session concludes (scaffolding is faded off). Considering that most studies did not measure the fade-off effect of AR apps on student mathematics performance (e.g., Arican & Özçakir, 2021), our finding provides a new insight in understanding the effects of AR apps for teaching area measurement. This finding provides empirical evidence of Bujak's et al. (2013) argument that AR helps students to retain acquired mathematical knowledge as AR can foster natural interaction, provide embodied representation, and enhance spatial understanding. We suggest further research into the short-term and long-term effects of AR activities on students' reasoning skills in teaching other geometric concepts such as length (one dimension) and volume (three dimension) measurements.

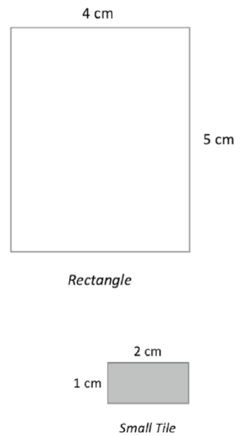
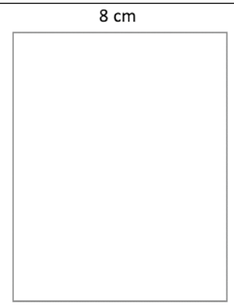
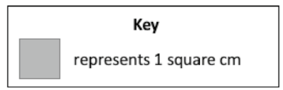
During the intervention stage, students improved their reasoning skills for both basic and composite shape problems. They employed effective strategies, such as decomposing and recomposing composite shapes, as facilitated through the use of the app (see Fig. 5). These findings validate previous studies that focused on basic shapes (Arican & Özçakir, 2021; Chao & Chang, 2019; Demitriadou et al., 2020), extending them by confirming that engagement with AR activities contributes to enhanced reasoning skills for both types (basic and composite shapes) of area problems. We suggest further research to examine the impact of AR activities on different types of area tasks, such as the conservation of area and the area of various 2D shapes.

We also observed variations among students in the speed of their developmental progressions in area measurement. Daisy showed an immediate jump in developmental progression, while the other two students showed relatively gradual progress. We anticipated that their initial reasoning skills and mathematics knowledge in other domains might influence these differences. However, all students reached the highest phase of the learning trajectory for area measurement with the help of our program. Thus, we argue that all students can construct area measurement knowledge and reasoning skills when they are provided a carefully designed intervention.

In particular, we found the synergistic value of technological and social scaffolding (Pea, 2018). Previous scholars have focused on either technological affordance (e.g., Fernández-Enríquez & Delgado-Martín, 2020) or instructional strategies of teachers (e.g., Sevinc & Brady, 2019) for students' learning of mathematics. However, this study addressed a gap identified in previous literature, finding complementary roles for mathematics instructors and technological devices.

In our study, instructors guided students to interact with virtual 2D units, enabling them to understand the structural elements of a 2D space and its relationships without needing to memorize an area formula. We concluded that verbal interactions with instructors during the AR activities played a beneficial role in their reasoning skills for area measurement. In this regard, our study contributes to the field by shedding light on the synergistic roles of an AR app and instructors in enhancing students' reasoning skills in mathematics. This finding opens the possibility of further research on the

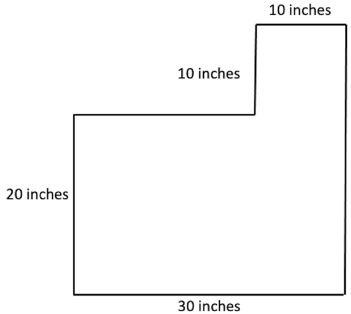
Table 12 Sample basic shape (rectangle) problems

Sample problem 1. How many small tiles would need to cover the rectangle? Show or explain your reasoning.	Sample problem 2. The area of a rectangle is 80 square centimeters. What is the length of the rectangle? Show or explain your reasoning.	Sample problem 3. Draw two different rectangles, each with an area of 16 squared centimeters. Show or express your reasoning.
		

Note. The first, second, and third problems were adopted from Clements et al. (2017), Barret et al. (2017), and Boaler (2015), respectively

Table 13 A sample composite shape (L-shape) problem

Sample problem 4. Find the area of a diagram. Show or explain your reasoning.



Note. The left and right problems were adopted from Clements et al. (2017) and Barret et al. (2017)

mediating role of instructors in an instructor-student-emerging technology interaction. These findings, which reflect outcomes from three students, offer a strong foundation for future investigation across larger sample volumes.

Conclusion

Success in children's learning of area measurement is essential, as it provides foundational skills in STEM throughout K–16 and beyond. Given the low achievement of children regarding this skill, this study developed an AR app, MeoGeo, and instructed students with it. Students were empowered by technological and social scaffoldings and showed the development of area measurement reasoning skills. Overall, this study provides a holistic picture of the AR activities process, involving not only the design of an AR app itself but also the role of instructors and the lasting effect of their synergistic roles. The success of the AR activities in this study offers a strong foundation for future investigations with broader scopes. Throughout this process, educators and researchers in both mathematics education and educational technology should collaborate to identify the best teaching practices.

Appendix

Table 12 represents three sample basic shape problems. The first sample problem, adapted from Clements et al. (2017), asked students to determine how many small tiles (1×2 small gray-colored tiles) would be needed to cover a 4×5 rectangle. Students could employ various strategies to solve this problem: Strategy (a), drawing small tiles inside a rectangle and counting their number (Phase 5, area unit relater and repeater); Strategy (b), calculating the area of each tile and a rectangle, then dividing the area of the rectangle by the tile's area (Phase 7, area row and column structurer); or Strategy (c), recognizing that five units of size 1 (the height of a small tile) makes 5, that two units of size 2 (the base of a small tile) makes 4, and that the total number of small tiles needed for the rectangle can be determined by multiplying 5 and 2 (Phase 8, array structurer). Strategies (b) and (c) show a higher developmental phase in reasoning area measurement than strategy (a), as those strategies demonstrate an efficient use of computing area based on a multiplicative reasoning.

The second sample basic shape problem, adapted from Barrett et al. (2017), provided students with a rectangle with a known base length of 8 cm, asking them to determine its height, given that the area was specified as 80 cm^2 (8 cm $\times$ $\square$ cm = 80 cm^2). This problem aimed to assess whether students considered spatial dimensions and the relationship between the base and height in area calculation (Phase 8, array structurer). Students who have a limited understanding on area measurement or only memorize the area formula may experience challenges to find the unknown factor.

The third sample basic shape problem, adapted from Boaler (2015), asked students to draw two different rectangles, each with an area of 16 cm^2. Rectangles with an area of 16 cm^2 can take the form of a 1×16 rectangle, a 2×8 rectangle, or a 4×4 rectangle, among others. This problem was designed to assess whether students understand concepts related to spatial dimensions and relationships between structural elements (units, rows, and columns) rather than merely applying a formula to calculate area (Phase 8. Array structurer).

The composite (L-shape) problem, adapted from Lehmann (2023), assesses whether students decompose a composite shape into two rectangles (see Table 13). For instance, they may consider decomposing it into a 10×10 rectangle and a 20×30 rectangle, or a 20×20 rectangle and a 10×30 rectangle. Students calculate the area of each basic shape and add them to find the total area, which is 700 cm^2.

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