MATHEMATICS TEACHER
Learning & Teaching
PK–12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

GPS: Artificial Intelligence Image Processing

AUTHOR NAMES:

Flavin, Eunhye; Lee, Jane HyoJin; Chamberlin, Michelle T.; and Powers, Robert A.

DIGITAL OBJECT IDENTIFIER: 10.5951/MTLT.2024.0103

VOLUME: 117

ISSUE NUMBER: 11

Mission Statement

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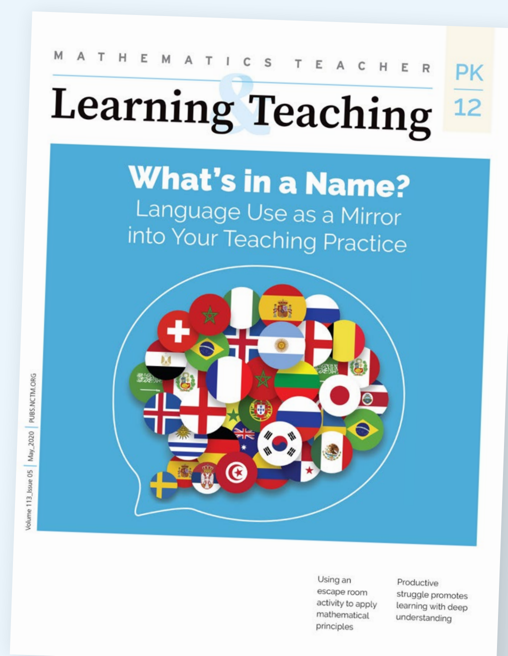
Approved by the NCTM Board of Directors on July 15, 2017.

CONTACT: mtlt@nctm.org



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GPS: Artificial Intelligence Image Processing

Growing Problem Solvers provides four original, related, classroom-ready mathematical tasks, one for each grade band. Together, these tasks illustrate the trajectory of learners' growth as problem solvers across their years of school mathematics.

Eunhye Flavin, Jane HyoJin Lee, Michelle T. Chamberlin, and Robert A. Powers

Mathematics propels the innovations in artificial intelligence (AI). In November's Growing Problem Solvers, we focus on matrix in the context of image recognition and processing. A matrix is a data structure: a set of elements, including numbers, symbols, and expressions, arranged in a rectangular array of rows and columns. A digital image is a "data structure within the computer, containing a number or code for each pixel or picture element in the image" (House, n.d, p. 1). For example, a 300 pixels × 200 pixels black-and-white image is equivalent to a 300 columns × 200 rows matrix. The code for each pixel determines the color of that pixel. AI technology stores image information and uses artificial neural networks to process images, including classification and pattern recognition. PK–12 students encounter numerous applications of AI-driven image recognition and processing in daily life, including biometric recognition on smartphones and self-checkout systems at grocery stores. However, they might not necessarily understand the mathematics that underlies such innovations. Students need to understand future-forwarding mathematics knowledge to critically participate in our increasingly AI-integrated society.

We introduce four interrelated mathematics tasks tailored for each grade band, where students explore how a computer recognizes and processes images through matrices. The term *matrix* is typically presented in high school in mathematics or science classes. However, the Common Core State Standards for Mathematics (CCSSM) includes mathematical concepts and practices that help to understand matrices throughout PK–12 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In Table 1, we identified the mathematics content and practice standards aligned with the four mathematical tasks we designed. PK–2 students learn how each entry of a matrix represents a pixel. Grades 3–5 learn how a matrix is used for storing images. Both the PK–2 and 3–5 tasks meet Mathematical Practice 7, as they ask students to find a pattern between a digital image and a corresponding array of 0s and 1s. Grades 6–8 develop basic transformation skills such as reflections with images in matrix form. Last, grades 9–12 explore how matrix operations such as matrix multiplication are used in image processing.

Digital color images consist of three layers of two-dimensional matrices. Each layer represents red, green, and blue. We aim to establish a groundwork for PK–12 students to understand matrices and their operation. Therefore, in each of the four tasks, we highlight a black-and-white image, which is a single layer of a two-dimensional matrix.

Table 1 CCSSM for the Tasks Presented in the Study

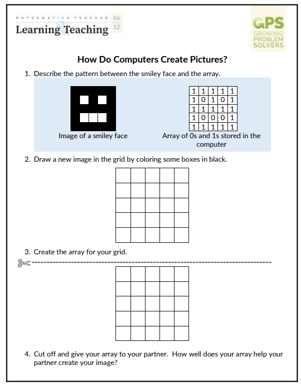
Grade Band	Mathematics Standards
PK–2	Mathematical Practice #7. Look for and make use of structure
3–5	Mathematical Practice #7. Look for and make use of structure
6–8	8.G.B.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
9–12	HSN-VM.C.8: Add, subtract, and multiply matrices of appropriate dimensions.

Note. National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010.

PK–2

In the PK–2 task, How Do Computers Create Pictures? (link online), students explore how a digital image is represented as a matrix with numerical values for computers to recognize and process. For Question 1, students begin with a 5×5 pixel image of a smiley face and the corresponding array of 0s and 1s on a 5×5 grid. Start by asking students to find a pattern between the image and the array. Mathematically, the key idea is that white pixels are represented by 0, and black pixels are represented by 1. If students do not recognize this pattern, ask them to count how many 1s are in the array and how that count might relate to the image.

Now, have students create their own image in the grid (Question 2) and then translate it to the array (Question 3). Students may need a reminder from the teacher that white and black pixels are represented by 0 and 1, respectively. Question 4 is an extension, in which students cut off their array with a pair of scissors and give it to a fellow student, so that their fellow student recreates the image based on the given array. Provide students with scissors and a blank 5×5 grid for recreating their partner's drawing. The student pairs then compare and discuss the original and recreated images.



Eunhye Flavin, eunhyeflavin@gmail.com, is a senior research scientist/research faculty member at Georgia Institute of Technology's Center for Education Integrating Science, Mathematics, and Computing. Her research focuses on pedagogical mediations, such as task design and technology development, to uplift historically marginalized populations in and through mathematics education.

Jane HyoJin Lee, she/her, jlee7@stonehill.edu, is an associate professor of Mathematics at Stonehill College in Easton, MA, where she teaches and conducts research in applied mathematics. She is also interested in secondary mathematics education.

Note. The first two authors contributed to this article equally.

Michelle T. Chamberlin, she/her, mchambe5@uwyo.edu, is an associate professor at the University of Wyoming, teaching mathematics and mathematics education classes for prospective and practicing teachers. She is interested in finding ways to enhance mathematics teacher education.

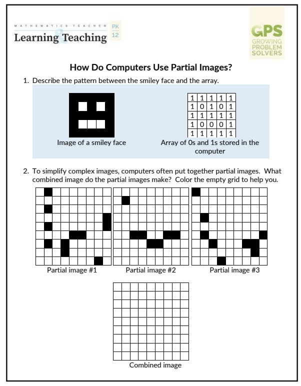
Robert A. Powers, he/him, robert.powers@unco.edu, is a professor of mathematical sciences at the University of Northern Colorado, teaching mathematics and mathematics education classes for preservice and in-service teachers. He is interested in task analysis for mathematics teacher educators.

doi:10.5951/MTLT.2024.0103

3–5

Computers use matrices (i.e., arrays of numbers) to handle digital images. Editing images on computers involves matrix computations. For instance, combining multiple images entails merging corresponding matrices through specific matrix operations. The grades 3–5 task, How Do Computers Use Partial Images? (link online), is designed to teach how computers use numbers in arrays to combine multiple images, focusing on the addition of 1s and 0s in three arrays to produce a single image. The key mathematical idea stays the same as the PK–2 task in that students look for the pattern between the colors of pixels and corresponding numerical values.

Before students begin Questions 2–4, teachers should discuss how a computer will recognize the pixels to efficiently figure out a whole picture, such as for facial recognition. On Question 2, each array on a 9 × 9 grid displays a fragmented portion of an image of a dinosaur, which is not immediately discernible.

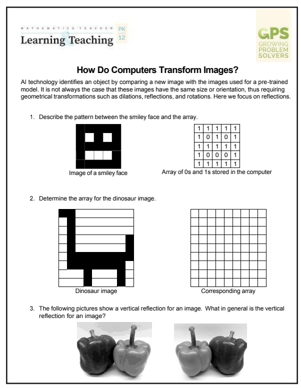


In Question 3, students write 0s or 1s in the corresponding positions of each array. In Question 4, they create a combined array that represents a whole picture (i.e., a dinosaur). This task helps students make sense of the role of the matrix as data storage. Questions 5–7 are an extension, in which students create their own image on a 9 × 9 grid, translate it into three partial arrays, and “transmit them” to a fellow student who recreates the image. The student pairs then discuss whether they were able to create the original image with the partial arrays.

6–8

AI technology identifies an object by comparing a new image with the images used for a pre-trained model. These images do not usually have the same size or orientation. Therefore, geometrical transformations—such as dilations, reflections, and rotations—are required. In the task for grades 6–8, How Do Computers Transform Images? (link online), students formalize the idea of vertical and horizontal reflections. For Questions 1–2, teachers can remind students how a digital image is understood as an array and then let students practice making a 9 × 9 array corresponding to 9 pixels × 9 pixels (i.e., the black-and-white dinosaur image).

In Questions 3–4, students first learn what vertical reflection means from the photo of peppers and then create a vertical reflection of the dinosaur using the array. If students struggle with Question 4, relate vertical reflections to a flip over the y-axis. Then ask, “What do you think a flip would do to the corresponding array?” Questions 5–6 expand students’ understanding of horizontal reflection. Questions 7–9 are extension activities, in which students create their image on a 9 × 9 grid and practice vertical and horizontal reflections. Have student pairs recreate the image from one another’s array and determine which reflection was applied to the partner’s array.

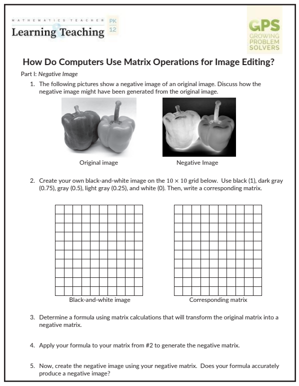


AI processes images using various matrix operations. In the task for grades 9–12, How Do Computers Use Matrix Operations for Image Editing? (link online), students experience how matrix addition and multiplication are used in image editing features.

In Part I: Negative Image, Question 1 invites students to discuss how a negative image might have been generated from the original image. In Question 2, teachers ask students to create their black-and-white image using five-shade scales on a 10 × 10 grid: black (1), dark gray (0.75), gray (0.5), light gray (0.25), and white (0) (see Figure 1). Teachers can provide the grid to students using online document editors (e.g., Microsoft Word), where students can electronically shade the cells with the given gray tones. In Question 3, students may need help to derive the formula to obtain the negative image matrix: Negative image matrix = Same size matrix with 1s in all entries – Original Image matrix. For example, the following equation corresponds to the shading in Figure 1. On Questions 4–5, students apply the formula to generate the negative matrix and create the negative image.

$$\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+(-1)\begin{bmatrix}1 & 0.75\\0.5 & 0.25\end{bmatrix}=\begin{bmatrix}0 & 0.25\\0.5 & 0.75\end{bmatrix}$$

In Part II: Mirror Image, students learn how the original matrix and the mirrored matrix are related in the context of matrix multiplication. Questions 1–3 teach how a matrix changes when a vertical reflection is applied. Teachers will introduce matrix multiplication with an identity matrix (see Equation 1). Then, have students explore how the identity matrix can be altered to produce a mirror image (Figure 2). Question 4 introduces the notion that matrix multiplication is not commutative (see Equations 2 and 3). Next, students engage in how to achieve a horizontal reflection of an image using matrix arithmetic. Teachers then facilitate a discussion about how changing the order of matrix multiplication would result in an upside-down image (Figure 3), instead of a mirror image. On Question 5, the task concludes with students creating an upside-down version of their vertically reflected image generated in Part I.


MATHEMATICS TEACHER Learning & Teaching PK–12

GPS GROWING PROBLEM SOLVERS

How Do Computers Use Matrix Operations for Image Editing?

Part I: *Negative Image*

1. The following pictures show a negative image of an original image. Discuss how the negative image might have been generated from the original image.

Original image — Negative Image

2. Create your own black-and-white image on the 10 × 10 grid below. Use black (1), dark gray (0.75), gray (0.5), light gray (0.25), and white (0). Then, write a corresponding matrix.

Black-and-white image — Corresponding matrix

3. Determine a formula using matrix calculations that will transform the original matrix into a negative matrix.

4. Apply your formula to your matrix from #2 to generate the negative matrix.

5. Now, create the negative image using your negative matrix. Does your formula accurately produce a negative image?

Equation 1

$$\underbrace{\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}}_{=A}\underbrace{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}}_{=I}=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} \qquad (1)$$

Equation 2

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}=\begin{bmatrix}3 & 2 & 1\\6 & 5 & 4\\9 & 8 & 7\end{bmatrix} \qquad (2)$$

Equation 3

$$\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}=\begin{bmatrix}7 & 8 & 9\\4 & 5 & 6\\1 & 2 & 3\end{bmatrix} \qquad (3)$$

Figure 1 Example of Shading of the Original Image (Left) and Its Negative (Right)

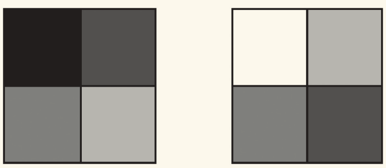


Figure 2 Mirror Image

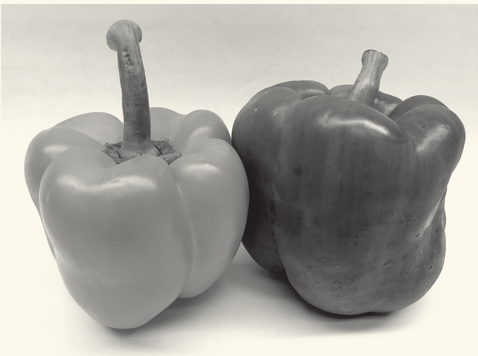


Figure 3 Upside-Down Image

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